Problem #107

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Date: 2010

Summary: Give a complete (resource free) characterisation of rewrite systems with polynomial derivational complexity.

It is well-known that well-founded monotone algebras form a complete characterisation for termination while such a result is currently unknown for polynomial derivational complexity. The notion of *resource freeness* is borrowed from implicit computational complexity theory. Here it refers to characterisations devoid of *direct* references to polynomial derivational complexity.

Currently suitably restricted matrix interpretations (see [MSW08, Wal10, NZM10]) form *the* method for proving polynomial upper bounds on the derivational complexity. Thus it is perhaps important to emphasise that matrix interpretations as studied in [EWZ08] are not sufficient as a starting point to solve the problem. Consider the one-rule TRS $\mathsf{g}(x, x) \to \mathsf{g}(\mathsf{a}, \mathsf{b})$. This TRS has linear derivational complexity, but no compatible matrix interpretation can exist.

Bibliography

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January 22, 2014