Problem #106

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Summary: Can we use the dependency pair method to prove relative termination?

The key of the success of the dependency pair method in proving termination is the following property from [AG00, GTSKF06], stated in more recent terminology:

> A TRS R is terminating if and only if the dependency pair problem $(DP(R), R)$ is terminating.

A dependency pair problem is a pair (P, R) of TRSs. Such a dependency pair problem is called terminating if it admits no infinite chain, that is, there is no $P \cup R$ reduction containing infinitely many P-steps, where P-steps only occur at the root.

Can we use the dependency pair method to prove relative termination? Here for a pair (R, S) of TRSs, R is said to be terminating modulo S if there is no $R \cup S$ reduction containing infinitely many R-steps. This is the same requirement as for termination of a dependency pair problem, except that the first TRS in a dependency pair problem may only be used for root steps. So, more precisely, the open problem is:

> Find a "useful" effectively computable function ϕ from pairs of TRSs to dependency pair problems, such that for every two TRSs R, S the TRS R is terminating modulo S if and only if the dependency pair problem $\phi(R, S)$ is terminating.

Here, "useful" means that the resulting dependency pair problem $\phi(R, S)$ should be "easy" (i.e., suitable for automated termination analysis by existing tools).

Bibliography

[AG00] Thomas Arts and Jürgen Giesl. Termination of term rewriting using dependency pairs. *Theoretical Computer Science*, 236:133–178, 2000.

[GTSKF06] Jürgen Giesl, René Thiemann, Peter Schneider-Kamp, and Stephan Falke. Mechanizing and improving dependency pairs. *Journal of Automated Reasoning*, 37(3):155–203, 2006.

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