

## Problem #104 (Solved !)

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*Summary: Termination of replacing two successive occurrences of the same symbol in a string*

Start by a finite string over the alphabet  $\{a, b, c\}$ . As long as two consecutive symbols are the same, they may be replaced by the other two symbols in alphabetic order. So

- $aa$  may be replaced by  $bc$ ,
- $bb$  may be replaced by  $ac$ , and
- $cc$  may be replaced by  $ab$ .

Can this go on forever?

This problem coincides with establishing termination of the string rewrite system consisting of the three rules

$$aa \rightarrow bc$$

$$bb \rightarrow ac$$

$$cc \rightarrow ab$$

Up to renaming it coincides with problem SRS/Zantema/z086 in the termination problem data base TPDB, on which all tools failed in the Termination Competition 2005. A variant of this problem on multisets, the Chamelon Problem, is known to be non-terminating.

### Remark

Termination of this system has been shown by Hofbauer and Waldmann [HW05]. The derivational complexity of this system is open, see Problem 105.

# Bibliography

- [HW05] Dieter Hofbauer and Johannes Waldmann. Termination of  $\{aa \rightarrow bc, bb \rightarrow ac, cc \rightarrow ab\}$ . Preprint, 2005.