Problem #101

Originator: Hitoshi Ohsaki [OT02]
Date: July 2002

Summary: Are universality and inclusion of AC-recognizable languages decidable?

An *AC-tree automaton* as defined by [Ohs01] is given by a signature Σ, a set of AC-axioms (that is, associativity and commutativity) for some function symbols of Σ, and a set of rewrite rules R of the form

$$
\begin{aligned}
f(q_1, \ldots, q_n) &\rightarrow q && (1)\\
f(q_1, \ldots, q_n) &\rightarrow f(p_1, \ldots, p_n) && (2)\\
q &\rightarrow p && (3)
\end{aligned}
$$

where the q's and p's are state symbols. Such an automaton accepts a term t iff it rewrites t modulo the given AC-axioms to some final state. $L(A)$ denotes the language recognized by an AC-tree automaton A; a language L is called *AC-recognizable* iff $L = L(A)$ for some AC-tree automaton A.

Are the following questions decidable?

- *Universality*: Given an AC-tree automaton A, is $L(A)$ equal to the set of all ground terms over the given signature Σ?
- *Inclusion*: Given AC-tree automata A and B, is $L(A)$ a subset of $L(B)$?

It has been shown [OT02] that emptiness of AC-recognizable languages is decidable. Furthermore, as a consequence of the results of [ZL03], universality and inclusion are decidable if transition rules of the form $f(q_1, \ldots, q_n) \rightarrow f(p_1, \ldots, p_n)$ are not allowed (this is the sub-class of so-called *regular* AC tree-automata). However, both questions are still open in the general case.

Remark

The inclusion problem of AC-tree automata is undecidable [OTTR05]. Decidability of universality is still an open question.

Bibliography

[Ohs01] Hitoshi Ohsaki. Beyond regularity: Equational tree automata for associative and commutative theories. In Laurent Fribourg, editor, *15th International Workshop on Computer Science Logic*, volume 2142 of *Lecture Notes in Computer Science*, pages 539–553, Paris, France, September 2001. Springer-Verlag.

[OT02] Hitoshi Ohsaki and Toshinori Takai. Decidability and closure properties of equational tree languages. In Sophie Tison, editor, *Rewriting Techniques and Applications*, volume 2378 of *Lecture Notes in Computer Science*, pages 114–128, Copenhagen, Denmark, July 2002. Springer-Verlag.

[OTTR05] Hitoshi Ohsaki, Jean-Marc Talbot, Sophie Tison, and Yves Roos. Monotone AC-tree automata. In Geoff Sutcliffe and Andrei Voronkov, editors, *12th International Conference on Logic Programming and Automated Reasoning*, volume 3835 of *Lecture Notes in Artificial Intelligence*, pages 337–351, Montego Bay, Jamaica, December 2005. Springer-Verlag.

[ZL03] Silvano Dal Zilio and Denis Lugiez. XML schema, tree logic and sheaves automata. In Robert Nieuwenhuis, editor, *14th International Conference on Rewriting Techniques*, volume 2706 of *Lecture Notes in Computer Science*, pages 246–263, Valencia, Spain, June 2003. Springer-Verlag.

January 22, 2014