Assignment 1 - Geometric Optimization (0368-4144)

Due: November 12, 2013

Problem 1

Distance selection on the line. Let P be a set of n points in the real line (in general position), and let $1 \le k \le \binom{n}{2}$ be a given parameter. Find the k-th smallest distance among the pairs of points of P.

Use parametric searching, and give details concerning the implementation of the generic decision procedure (parallel algorithm, critical values, etc.) What is the running time of the algorithm?

Problem 2

Rectilinear 2-center: Let P be a set of n points in the plane. Using parametric searching, find two axis-parallel squares of smallest possible (equal) size, whose union contains P.

Give details concerning the decision procedure, generic implementation, etc. What is the running time of the algorithm? (**Note:** This is not the optimal way to solve the problem.)

(**Hint:** Exploit simple properties of the layout of the two covering squares.)

Problem 3

Let $x_1, \ldots, x_n$ be n distinct real numbers. We choose a random sample R of them of size r, by making r independent draws, each uniformly at random (so that in each draw each number can be chosen with probability $1/n$); note that we may have repetitions in R.

(a) Show that, with high probability, every interval between two consecutive elements of R, as well as the interval preceding the smallest one and the interval succeeding the largest one, contains at most $\frac{cn}{r} \log r$ of the original numbers x_i. (The probability increases with the constant c.)

(b) Consider the slope selection problem, for a set L of n lines and parameter k. Choose a random sample R of the $\binom{n}{2}$ vertices of the arrangement of L, as above, with $r = cn \log n$. Derive, using (a), a randomized algorithm that solves the slope selection problem in expected time $O(n \log^2 n)$ (without parametric searching).

Problem 4

1-center with outliers. Let S be a set of n points in the plane, and $k < n$ an integer.

Find the smallest disk that contains all but k points of S. Describe briefly an algorithm, based on parametric searching, that runs in roughly quadratic time.

Describe in more detail the parallel implementation of the decision procedure, which can be reduced to the following: Given n equal disks in the plane, find a point that lies in the maximum number of disks. (**Hints:** (i) Line sweeping is hard to parallelize, so avoid it. (ii) Show (and use) the fact that it suffices to look for the deepest point on the boundaries of the given disks.)

Problem 5

Minimum L_∞-Hausdorff distance under translation. Let A and B be two sets, each of n points in the plane, and let $\delta > 0$ be a given parameter. Describe a decision procedure for the following variant of the Hausdorff distance problem: Does there exist a translation t, such that for every point $a \in A$, $a + t$ lies at L_∞-distance $\leq \delta$ from some point of B (i.e., for each $a \in A$ there exists $b \in B$ such that their L_∞-distance is at most δ).

The running time should be $O(n^2 \log n)$.

(Recall that the L_∞-distance between a and b is max $\{|a_x - b_x|,\ |a_y - b_y|\}$. Some comments on this problem will be made in class.)