# COMPUTATIONAL GEOMETRY - FINAL EXAM

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**Answer four of the following six problems. All problems have equal weight (25 percent). You may use up to two sheets (A4) only, of any written material.**
**The exam is 3 hours long.**
**You may assume general position of the input.**
**Good luck!!**

## Problem 1

Let $A$ and $B$ be two sets of points in the plane, each consisting of $n$ points.
(a) Describe an efficient algorithm which finds, for each point of $A$, its nearest neighbor in $B$.
(b) Connect each point $a \in A$ to its nearest neighbor $N(a)$ in $B$ by a straight segment. Show that these segments do not cross each other.
(c) True or false? (i) All these edges are Delaunay edges of $A \cup B$. (ii) At least one of these edges is a Delaunay edge of $A \cup B$.

## Problem 2

Let $A$ and $B$ be two sets of points in three dimensions, each consisting of $n$ points. Suppose that $A$ and $B$ are separated by the $xy$-plane; that is, all the points of $A$ lie above the plane and all the points of $B$ lie below it. We want to find the nearest pair of points $a \in A$, $b \in B$; that is

$$d(a, b) = \min\{d(u, v) \mid u \in A,\ v \in B\}.$$

(a) Show that there exist a pair of parallel supporting planes $h_a$, $h_b$, to $A$ at $a$ and to $B$ at $b$, such that they separate $A$ and $B$, and $ab$ is perpendicular to both of them. Show also that the distance between $h_a$ and $h_b$ is the largest between any pair of parallel supporting separating planes of $A$ and $B$.
(b) Show that the problem of finding $a$ and $b$ can be expressed as a linear program with a convex objective function, and that it can be solved in $O(n)$ time.

## Problem 3

Let $P$ be a set of $n$ points in the plane. Preprocess $P$ into a data structure of quadratic size, so that, for any query point $q$ we can report, in $O(\log n)$ time, the number of points of $P$ at distance at most 1 from $q$. (**Hint:** Represent each point $p \in P$ by a disk of radius 1 centered at $p$.)

## Problem 4

Let $P_1, P_2, \ldots, P_{10}$ be 10 sets of points in $\mathbb{R}^3$, each consisting of $n/10$ points, so that all the points of $P_i$ lie on a common vertical line $\ell_i$ (parallel to the $z$-axis), for $i = 1, \ldots, 10$. For each $i$, the points of $P_i$ are given in their vertical order along $P_i$. Let $P = \bigcup_{i=1}^{10} P_i$.
(a) Show that $\text{Vor}(P)$ has linear complexity.
(b) Give a linear time algorithm for computing $\text{Vor}(P)$.
(**Hint:** First consider each of the individual diagrams $\text{Vor}(P_i)$, and then...)

## Problem 5

Let $e_1, \ldots, e_n$ be $n$ line segments in the plane, and let $r$ be a positive number. For each $i$, let $K_i$ be the expansion of $e_i$ by distance $r/2$, i.e., the set of all points at distance at most $r/2$ from $e_i$. What is the shape of each $K_i$?

Use the $K_i$'s to give an efficient algorithm for determining whether there exist a pair of segments so that the distance between them is smaller than $r$.

## Problem 6

Let $e_1, \ldots, e_n$ be $n$ line segments in the plane, all lying above the $x$-axis. Preprocess them into a data structure of quadratic size, so that, given any query ray $\rho$ emanating from a point on the $x$-axis, we can count, in $O(\log n)$ time, the number of segments $e_i$ that $\rho$ intersects. (The ray $\rho$ is specified by the pair $(x, \kappa)$, where $(x, 0)$ is its origin and $\kappa$ is its slope.)