Assignment 1 - Computational Geometry (0368-4211)

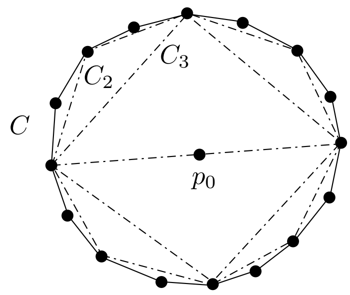
Due: April 13, 2015

Problem 1

Let C be a convex polygon with $n = 2^k$ vertices.

(a) Describe an algorithm, different from the one in (b), that uses $O(n)$ storage, so that, given a query point q, determines, in $O(\log n)$ time, whether q lies in C.

(b) Construct a sequence of sub-polygons as follows. $C_1 = C$. Suppose C_i has been constructed. Then C_{i+1} is constructed by taking every other vertex of C_i (the first, third, fifth, and so on), and by taking the convex hull of them.



Let p_0 be a point in the deepest polygon (which is in fact a line segment!). Given any query point q, we want to determine whether q lies inside C, by considering the segment p_0q, and by tracing it from p_0 to q, through the sequence of sub-polygons. (No additional data structures are needed.)

Explain how exactly the algorithm works, and show that it takes only $O(\log n)$ time. In fact, show that this algorithm can be applied even without explicit construction of the sub-polygons C_i.

Problem 2

Describe an $O(n \log n)$ incremental algorithm for computing $CH(P)$, for a set P of n points in the plane: Add the points of P one by one, and update the convex hull after each insertion in $O(\log n)$ time.

Problem 3

Let S be a set of n points in the plane. Show how to prepare a data structure that uses $O(n)$ storage, so that, given any query line ℓ, we can determine, in $O(\log n)$ time, (a) whether ℓ separates S (i.e., each of the halfplanes bounded by ℓ contains points of S), and (b) the point of S farthest from ℓ.

Problem 4

Let C_1 and C_2 be two convex polygons, each consisting of n edges. Give linear-time algorithms for the following problems:

(a) Determine whether C_1 and C_2 intersect.

(b) If they do intersect, compute the intersection $C_1 \cap C_2$ (of their interiors).

(c) If they do intersect, find the smallest translation that separates them. Formally, find a vector u, such that (i) $C_1 + u := \{x + u \mid x \in C_1\}$ and C_2 have disjoint interiors, and (ii) $\|u\|$ is the smallest possible.

Problem 5

(a) Let $\mathcal{D}$ be a set of n disks in the plane, all having the same radius. Describe the shape of the convex hull $CH(\mathcal{D})$, and the structure of its normal diagram, and give an $O(n \log n)$ algorithm for computing it.

(b) Let $\mathcal{D}$ be a set of n disks in the plane, with arbitrary radii. Describe the shape of the convex hull $CH(\mathcal{D})$, and the structure of its normal diagram, and show that the number of straight segments on its boundary is at most $2n - 2$.