Order out of Chaos: Proving Linearizability Using Local Views

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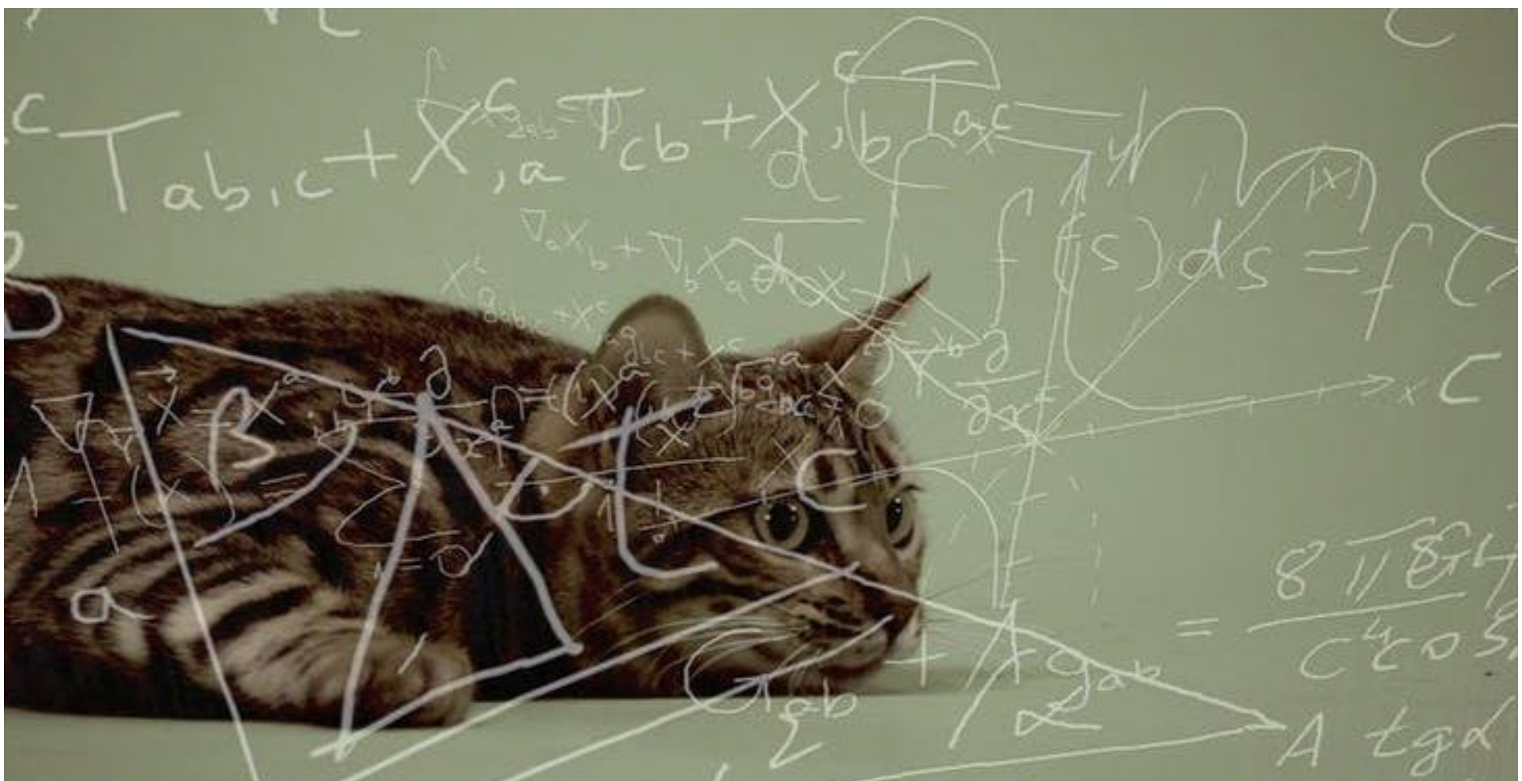
Concurrent Reasoning is Hard

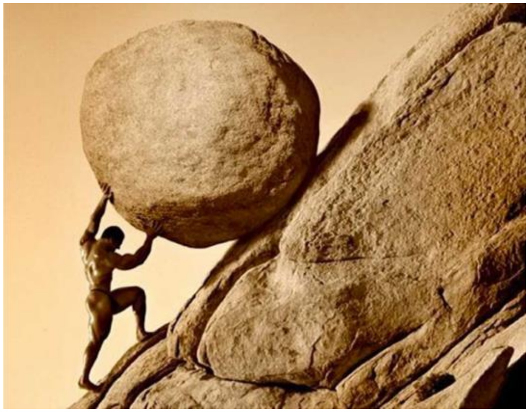
Goal: Prove linearizability of
highly-concurrent data structures



Optimistic Traversals

Good performance, hard proofs

- Lazy list [OPODIS’05]
- Lock-free skiplist [Fraser-2004]
- Contention-Friendly Tree [EuroPar’13, PPL’16]
- Lock free trees [PPOPP’14b, SPAA’12, PPOPP’14a, PODC’10, ICDCN’15]
- …



This Work

Main challenge: correctness of optimistic **traversals**

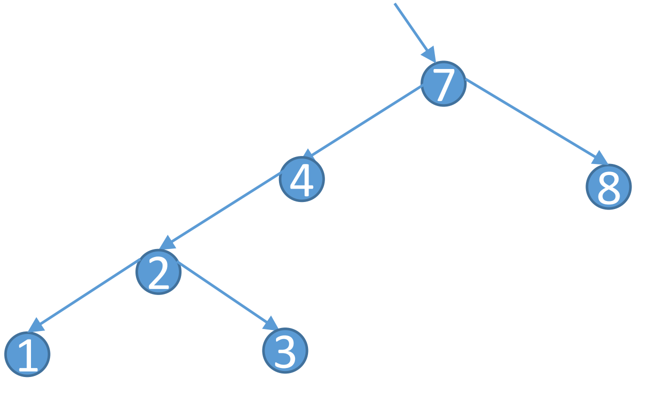
Traversal correctness **without interference** (sequential reasoning)

→

Traversal correctness **with interference** (concurrent reasoning)

Example: Lazy Binary Tree

- Variant of Contention-Friendly Tree [EuroPar'13, PPL'16]
- **Traversal does not take locks**
  - Used by contains, insert & delete
- Self balancing – rotation allocates a new node
- Marks logical & physical deletion by separate bits

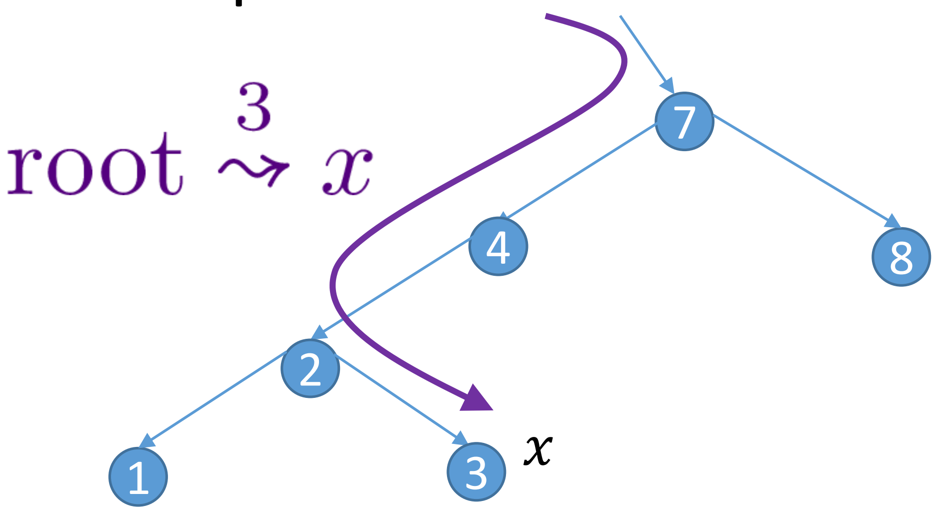


Linearizability Proof Uses Paths

- A tree represents a set using search paths:

$$\mathcal{A}(H) = \{k \in \mathbb{N} \mid \exists x.\ \mathrm{root} \overset{k}{\rightsquigarrow} x \wedge x.key = k \wedge \neg x.del\}$$

- Linearization point: set modified by at most one write in each operation



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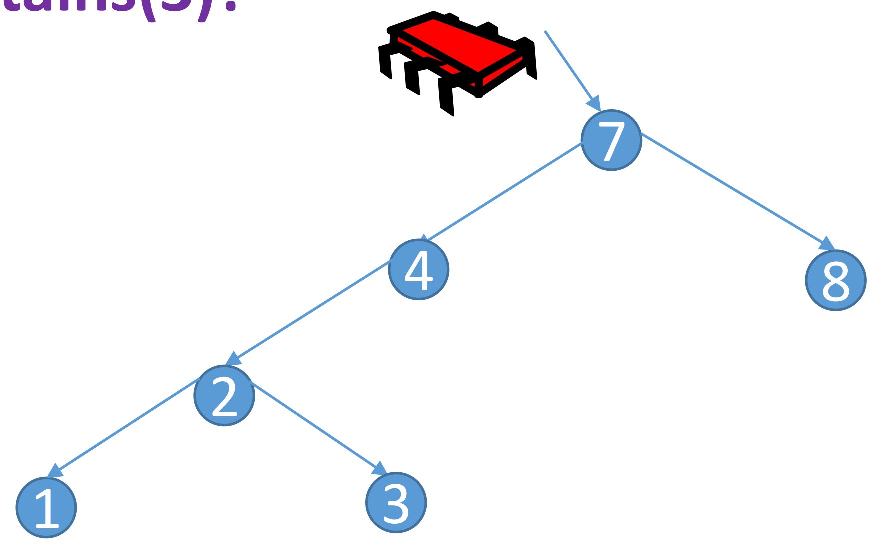
Correct **traversals** required
for correct paths manipulation

- **Contains:** exists path to key/null at some point
- **Insert/delete:** add/remove path
- Maintenance operations (e.g. rotate):
  no observable effect

Traversals: Inconsistent View

contains(3)?

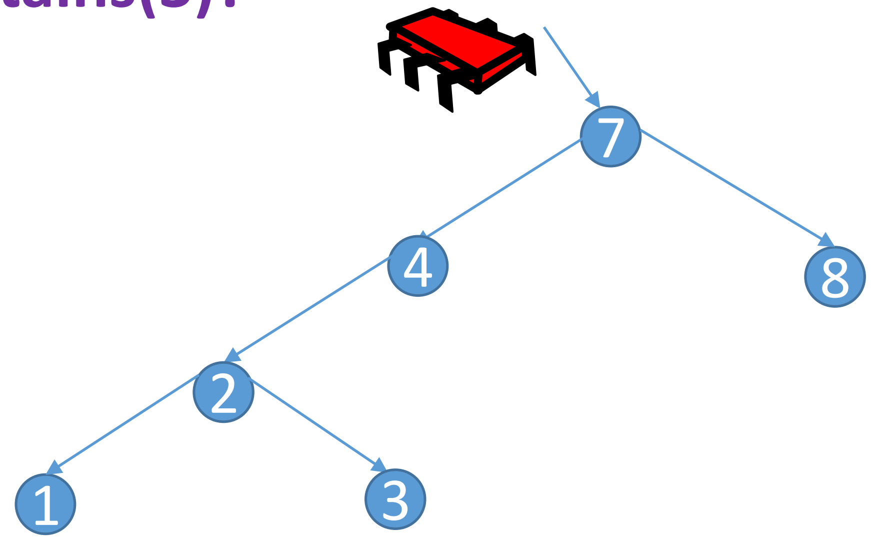


Traversals: Inconsistent View

thread's view

contains(3)?

global memory



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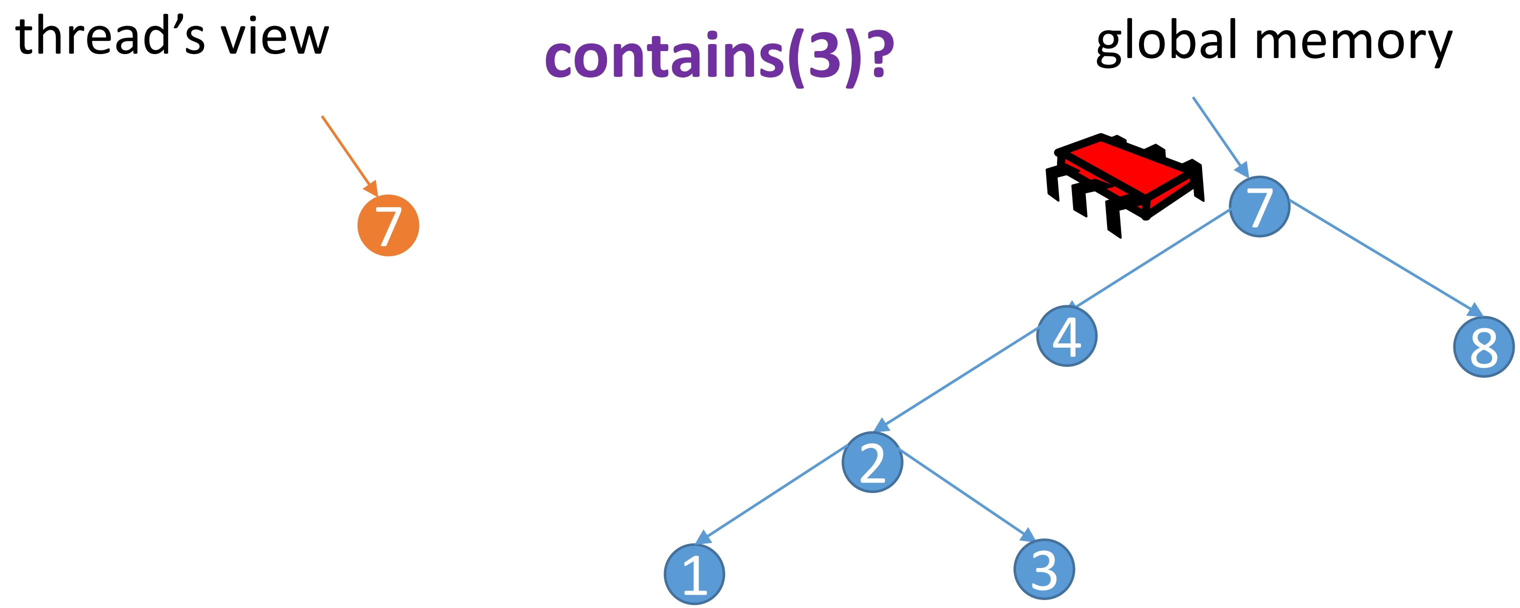

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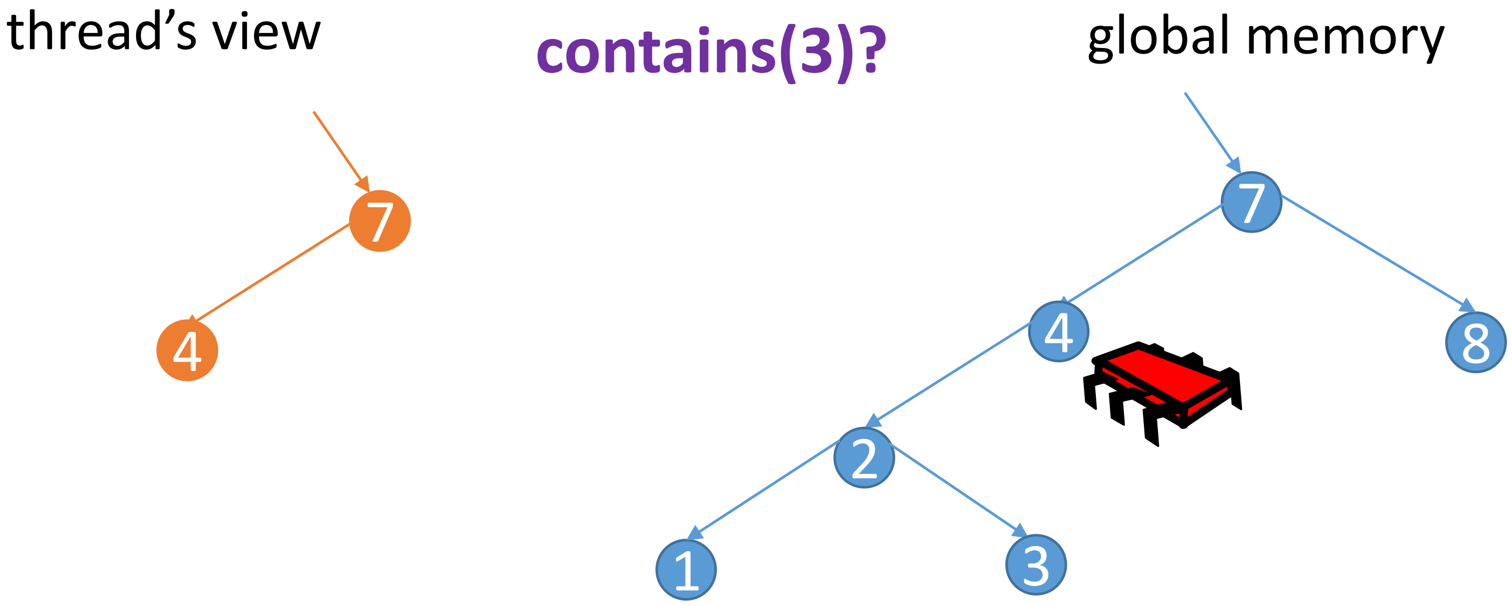

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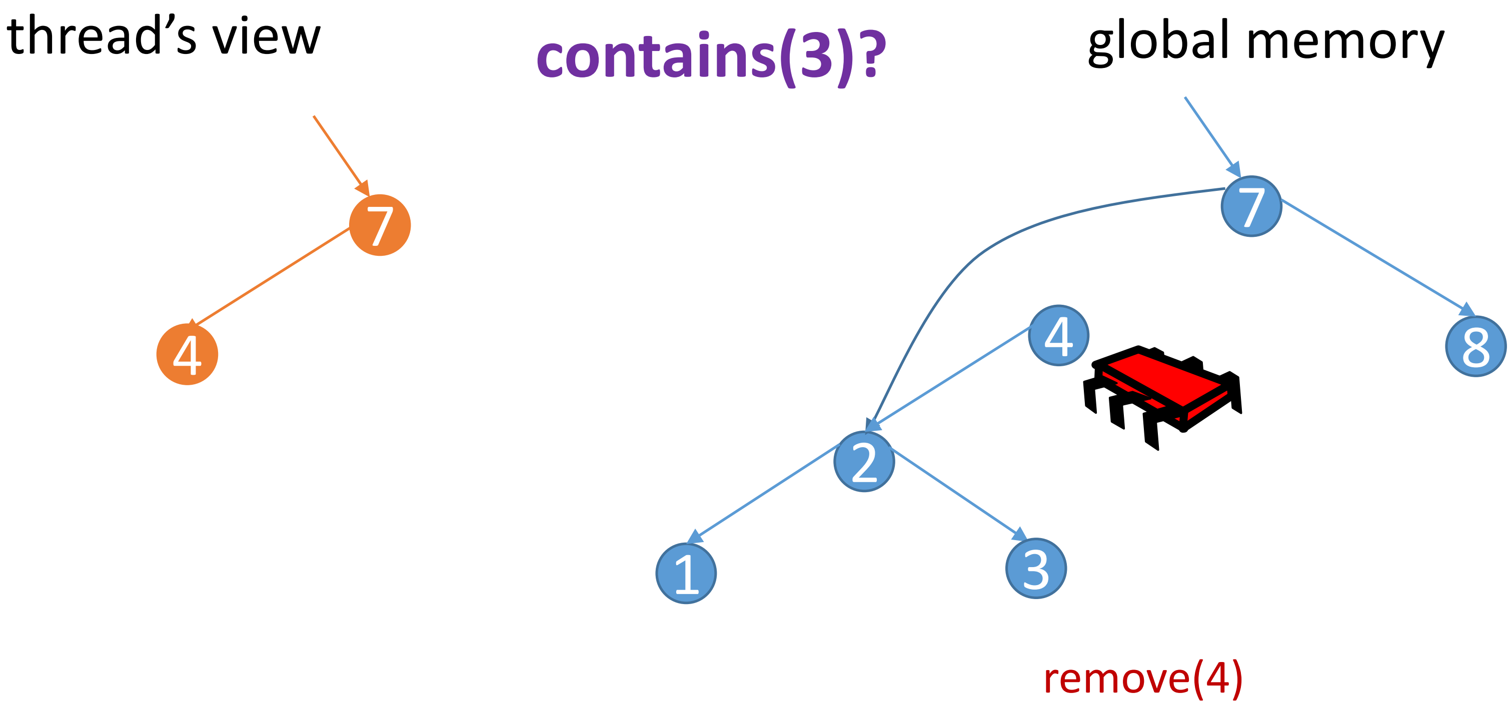

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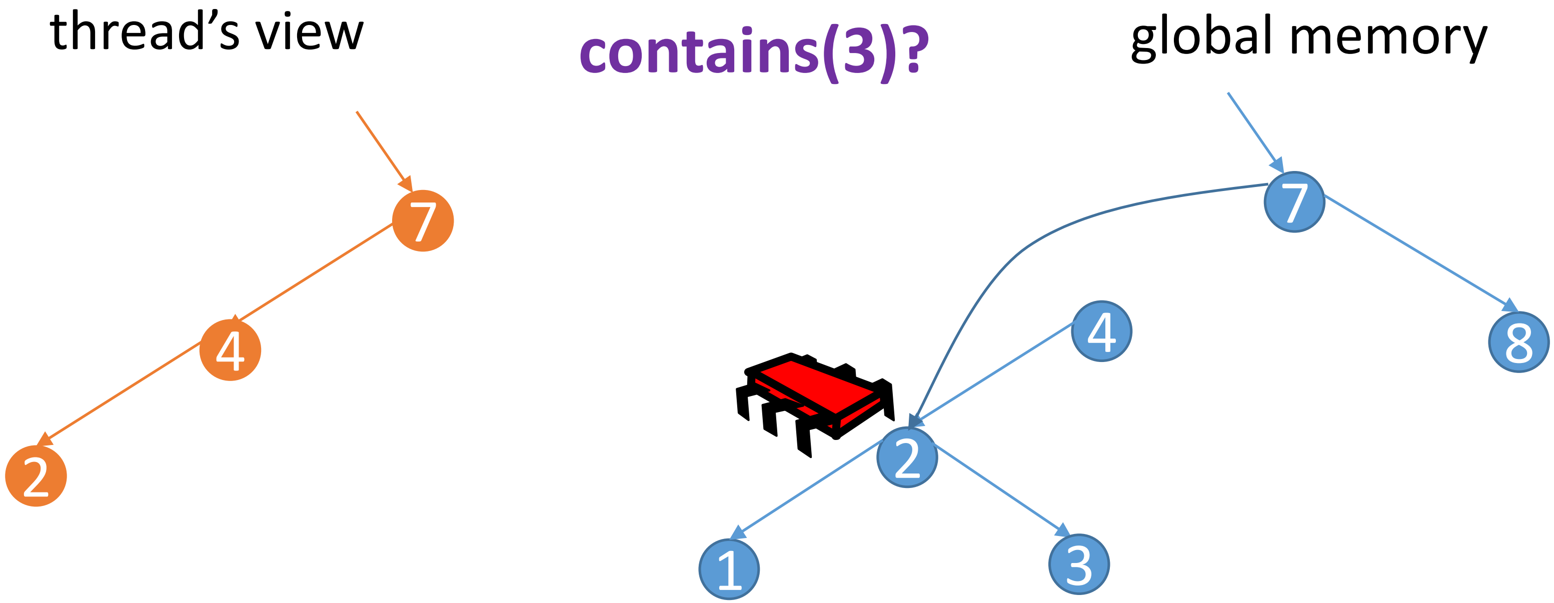

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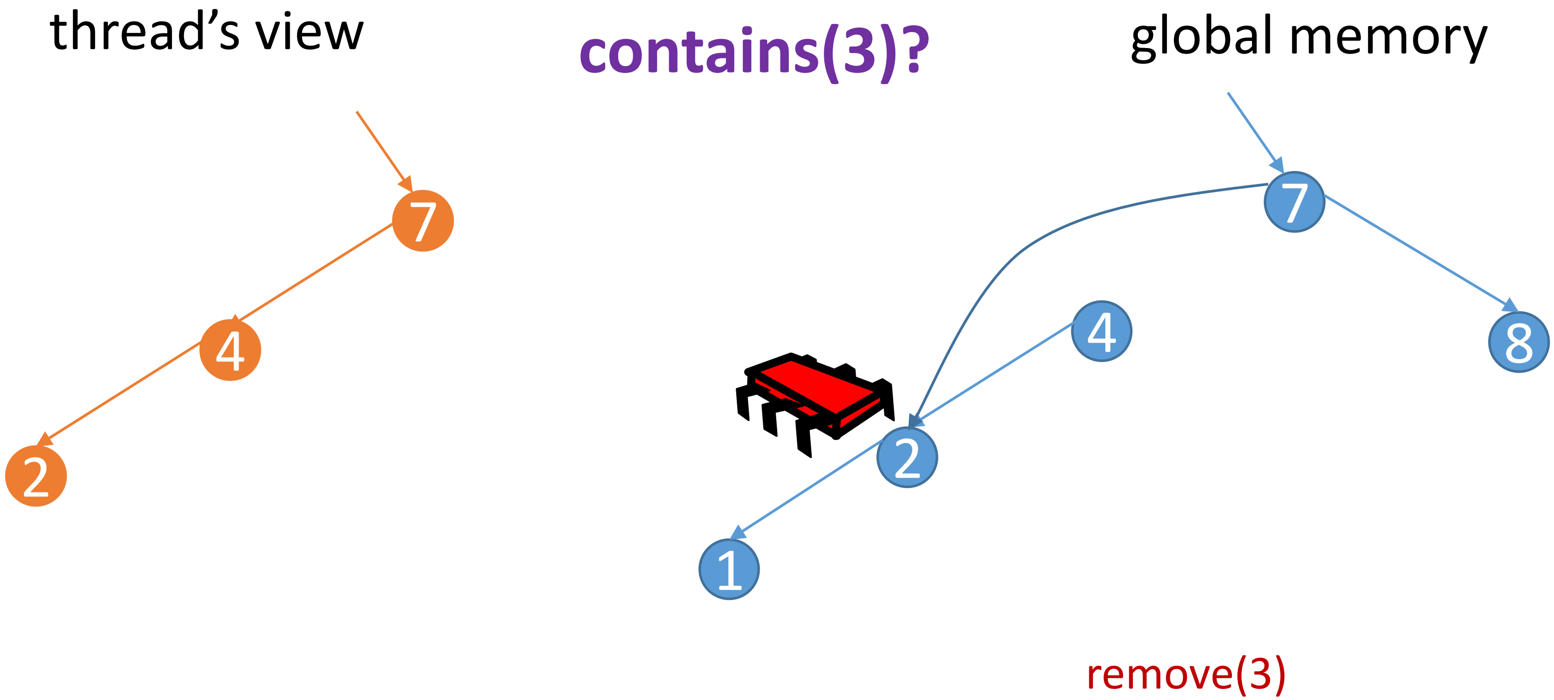

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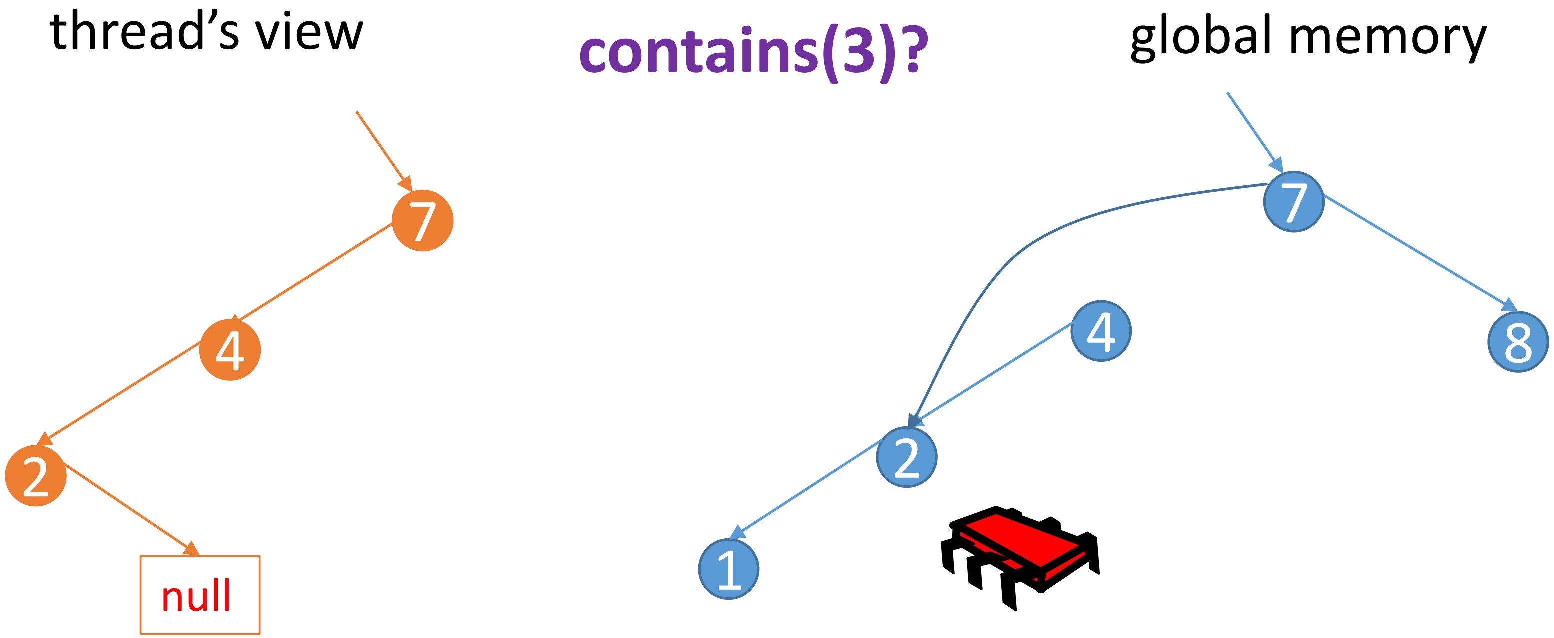
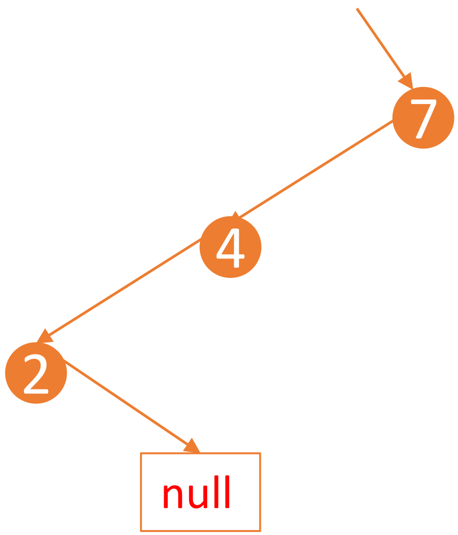


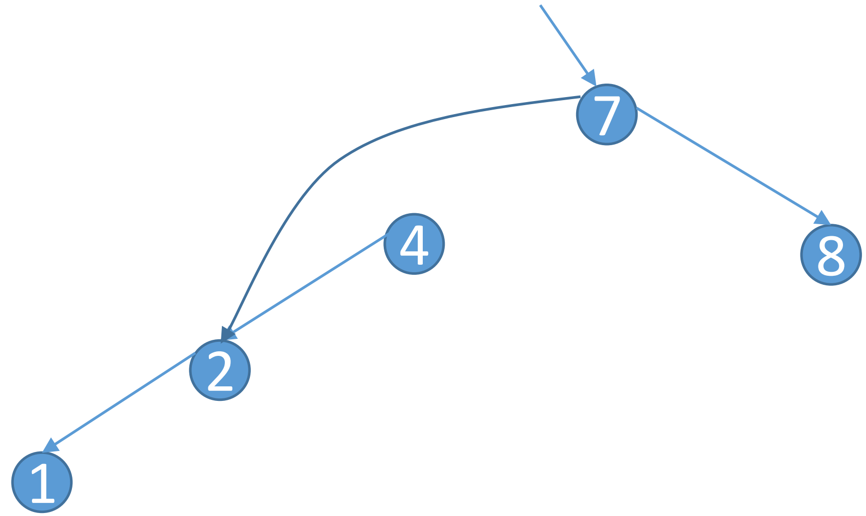
Traversals: Inconsistent View

thread's view

contains(3)? no

global memory





$$\text{root} \overset{k}{\rightsquigarrow} \mathit{null}$$

contains(3): **false**

$$\diamondminus\left(\text{root} \overset{k}{\rightsquigarrow} \mathit{null}\right)$$

contains(3): **false**

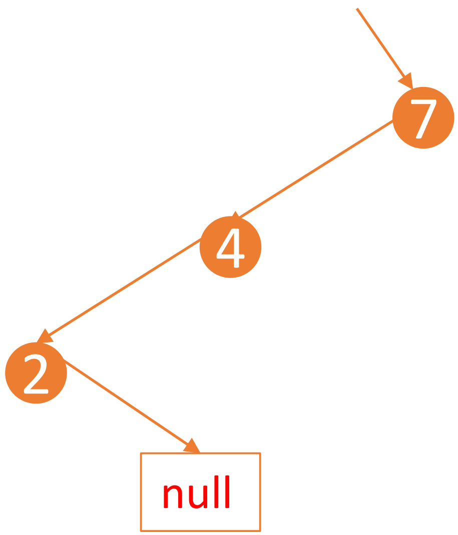
at some point

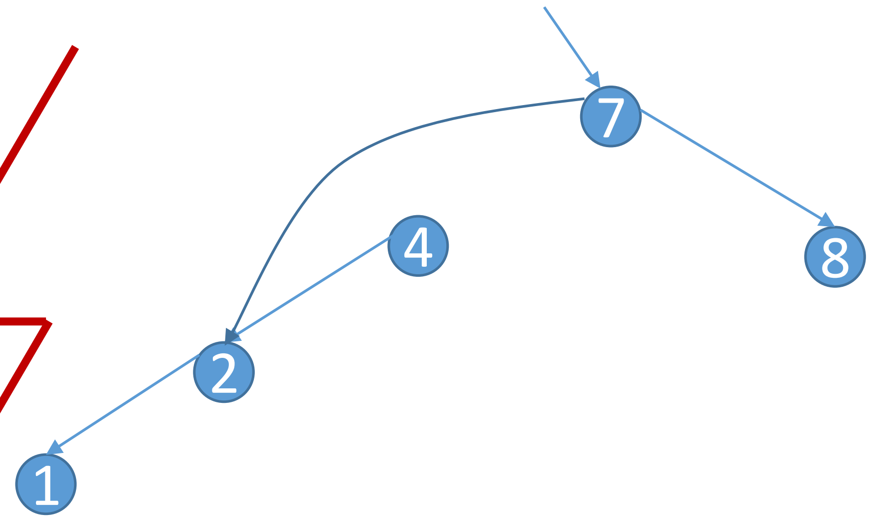
Traversals: Inconsistent View

thread's view

contains(3)? no

global memory





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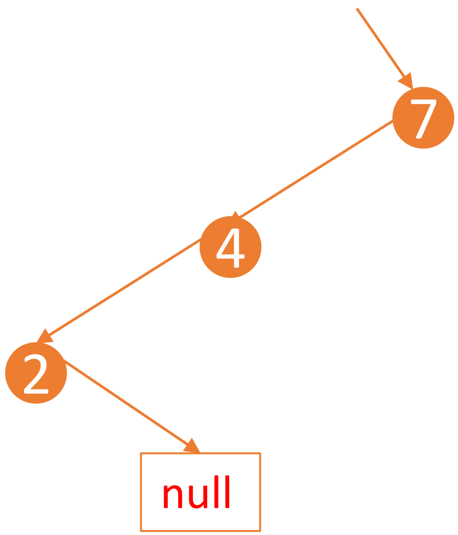

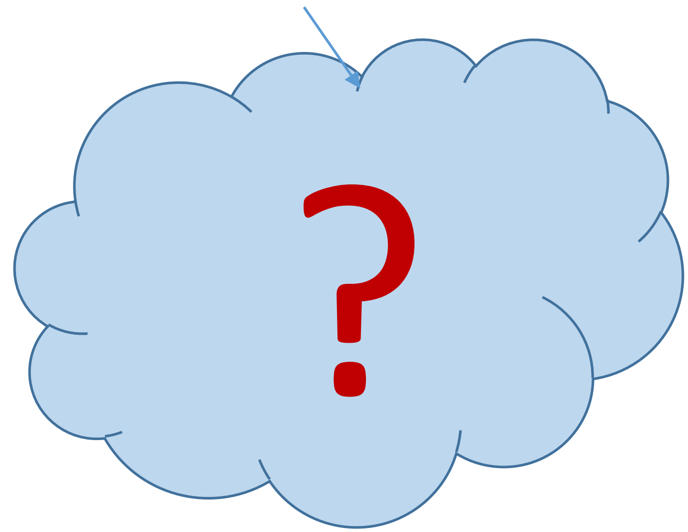

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contains(3): **false**

at some point

Traversal Correctness Problem

Prove the **existence** of a **search path**
in **real memory**
at some point during the traversal

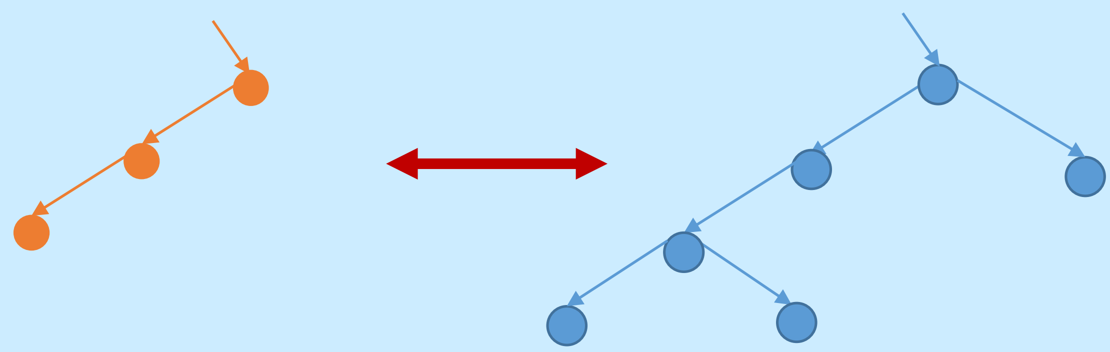
- The heart of the linearizability proof:
- **Contains:** $\diamondminus(\mathrm{root} \overset{k}{\leadsto} x)$
- **Insert:** $\diamondminus(\mathrm{root} \overset{k}{\leadsto} x)$+unmarked
- Full details in the paper

$\diamondminus(\mathrm{root} \overset{k}{\leadsto} x)$

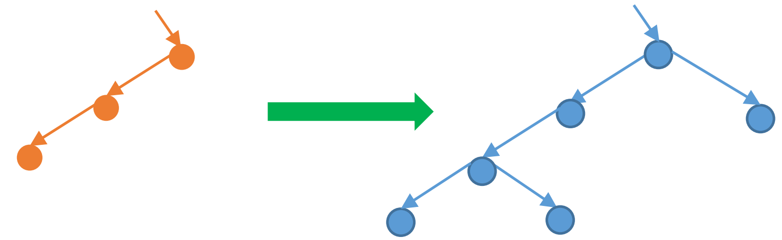
Traversal Correctness Problem

Prove the **existence** of a **search path**
in **real memory**
at some point during the traversal

How to prove traversal correctness?



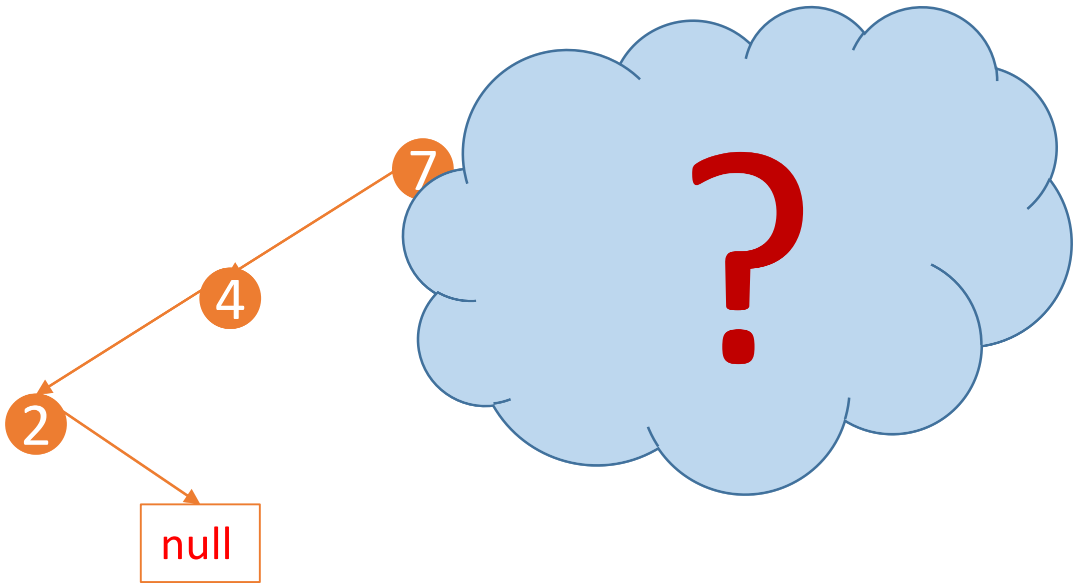
Our Approach



Traversal correctness **without interference** (sequential reasoning) → Traversal correctness **with interference** (concurrent reasoning)

- Apply the technique to prove the linearizability of
  1. Variant of Contention-Friendly self-balancing binary tree [EuroPar’13, PPL’16]
  2. Lazy list and Optimistic list [OPODIS’05]
  3. Lock-free list
  4. Lock-free skiplist

Local View Argument

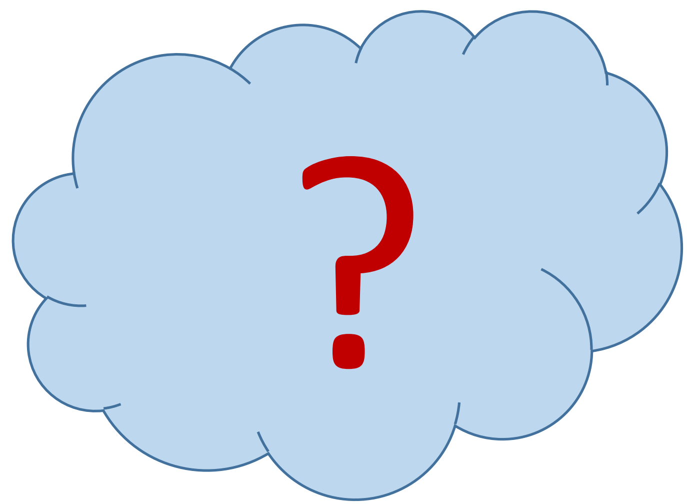


$$\Diamond(\mathrm{root} \overset{k}{\rightsquigarrow} x)$$

$$\mathrm{root} \overset{k}{\rightsquigarrow} x \quad \Longrightarrow$$

Local View Argument

Sequential reasoning to show traversal correct without interference



$$\mathrm{root} \overset{k}{\leadsto} x \quad \Longrightarrow \quad \diamondminus(\mathrm{root} \overset{k}{\leadsto} x)$$

Local View Argument

Sequential reasoning to show traversal correct without interference

Correctness of concurrent traversals

$$\mathrm{root} \overset{k}{\leadsto} x \quad \Longrightarrow \quad \Diamond(\mathrm{root} \overset{k}{\leadsto} \mathit{null})$$

Local View Argument

Order & Preservation

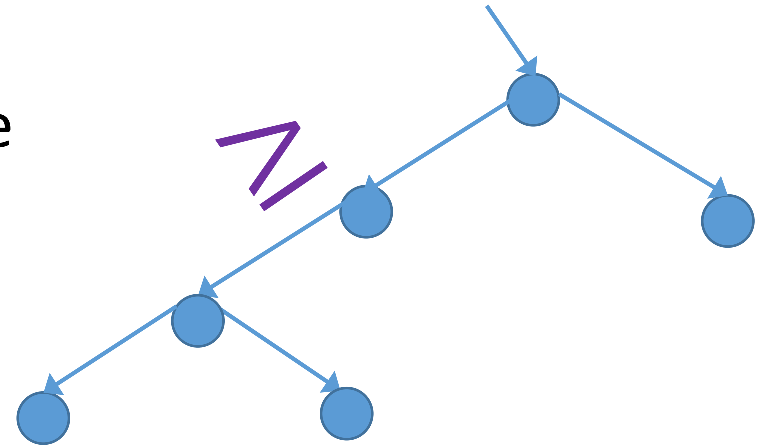
Sequential reasoning to show traversal correct without interference

Correctness of concurrent traversals

$$\mathrm{root} \overset{k}{\rightsquigarrow} x \quad \Longrightarrow \quad \diamondminus(\mathrm{root} \overset{k}{\rightsquigarrow} null)$$

Requirement 1: Order on Memory

- Partial order $\leq_H$ on **memory** induced by the **pointers** in the global state H (**not** order on the data)



- Reads follow the order
- Search paths follow the order
- **Temporal acyclicity**: the order is not violated across intermediate states

Requirement 1: Temporal Acyclicity

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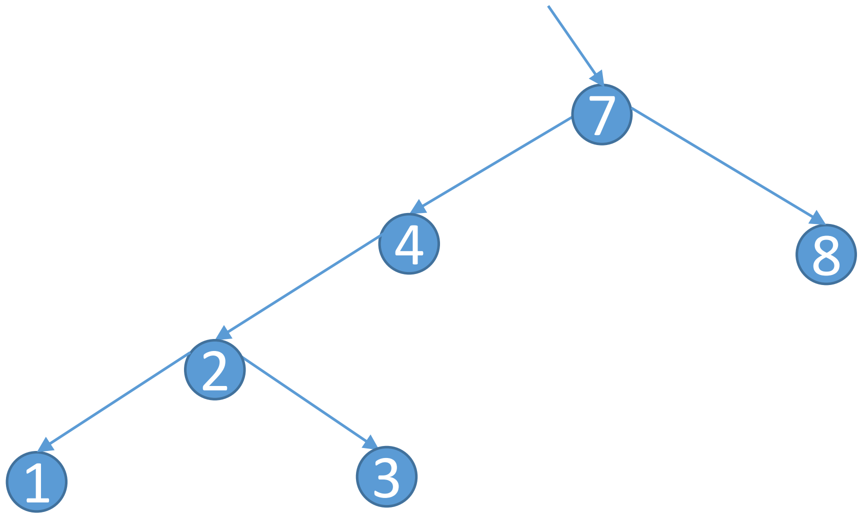

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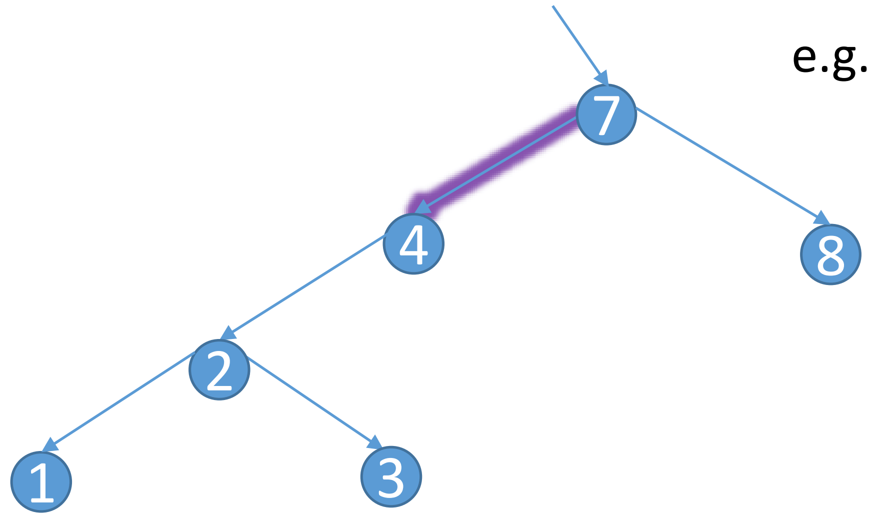

e.g. $\&4 > \&7$

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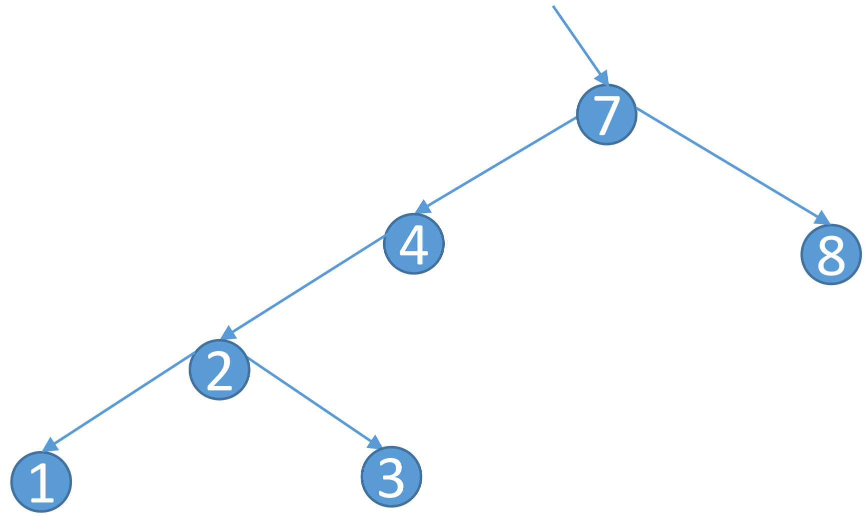

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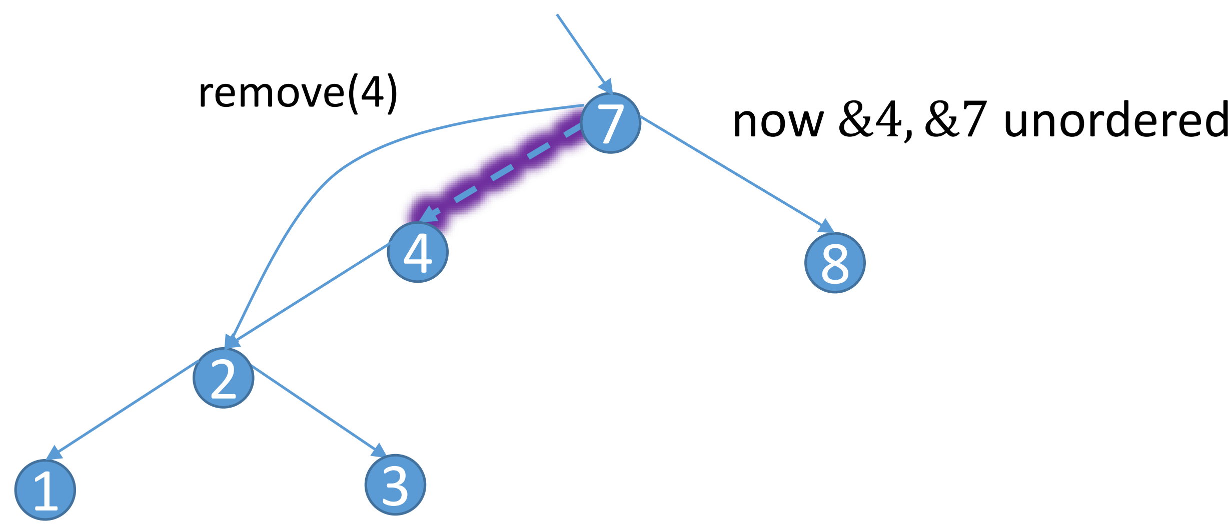

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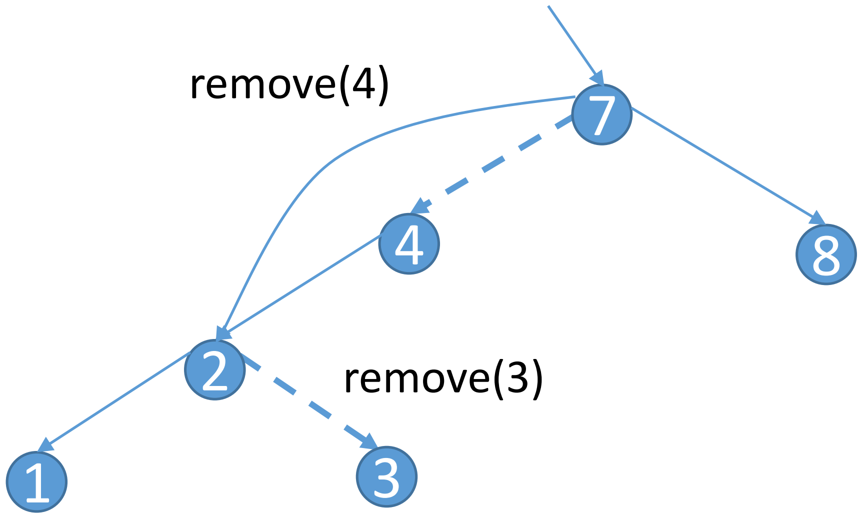

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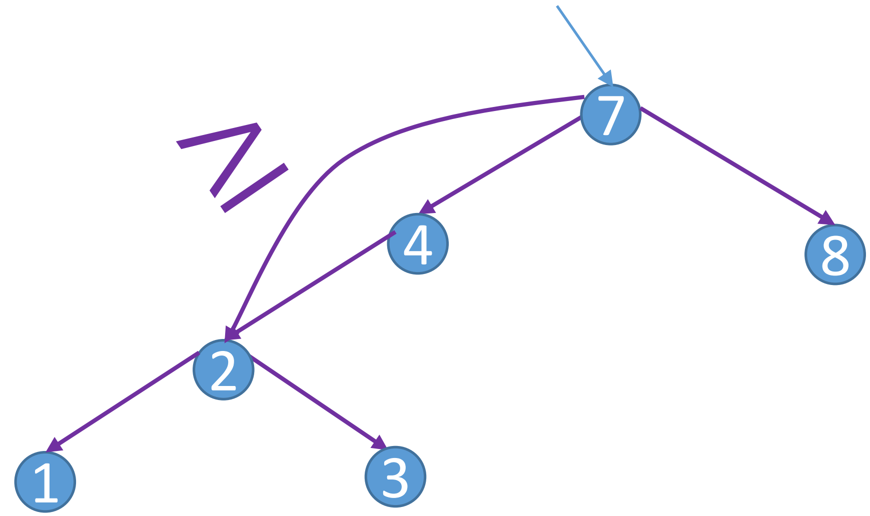
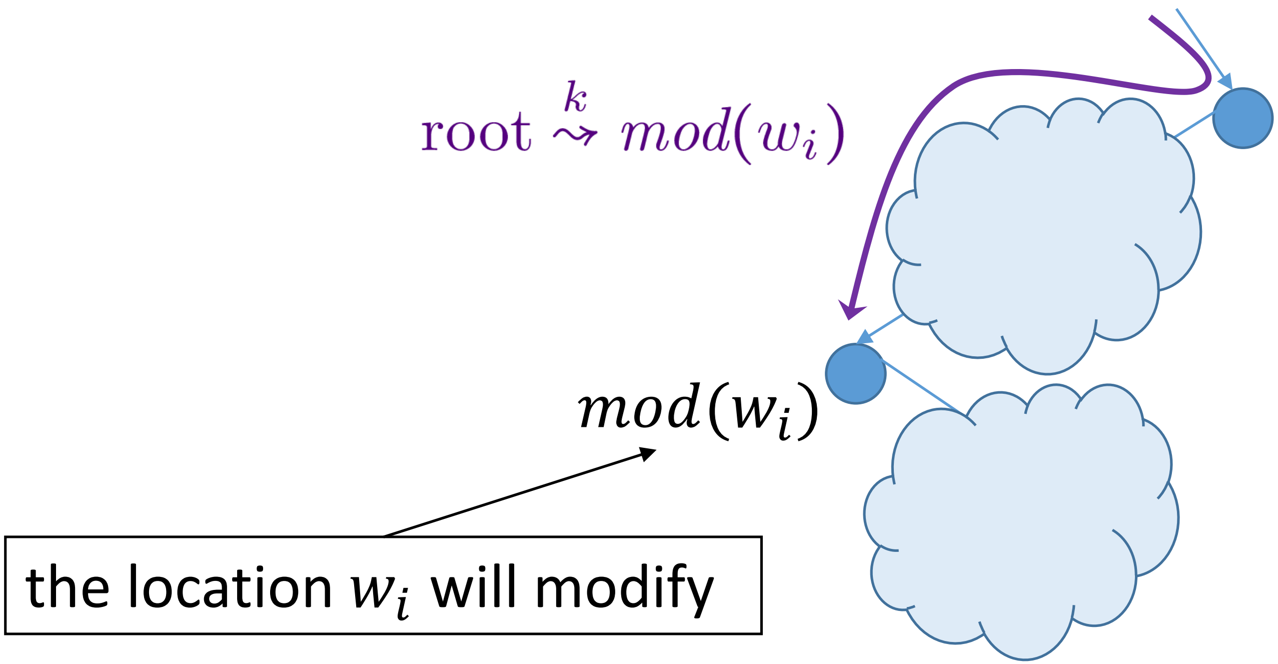


<u>Requirement</u>: no cycles in accumulated order

Requirement 2: Preservation

Writes do not destroy search paths to locations to which there could be later writes



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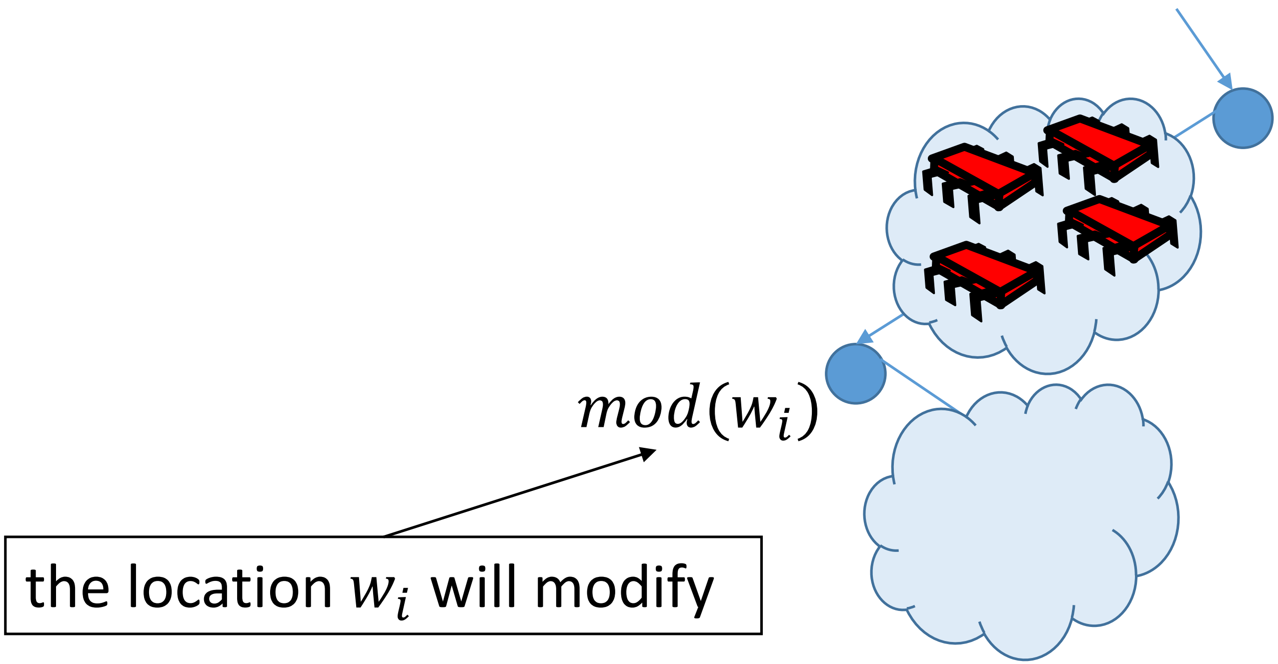

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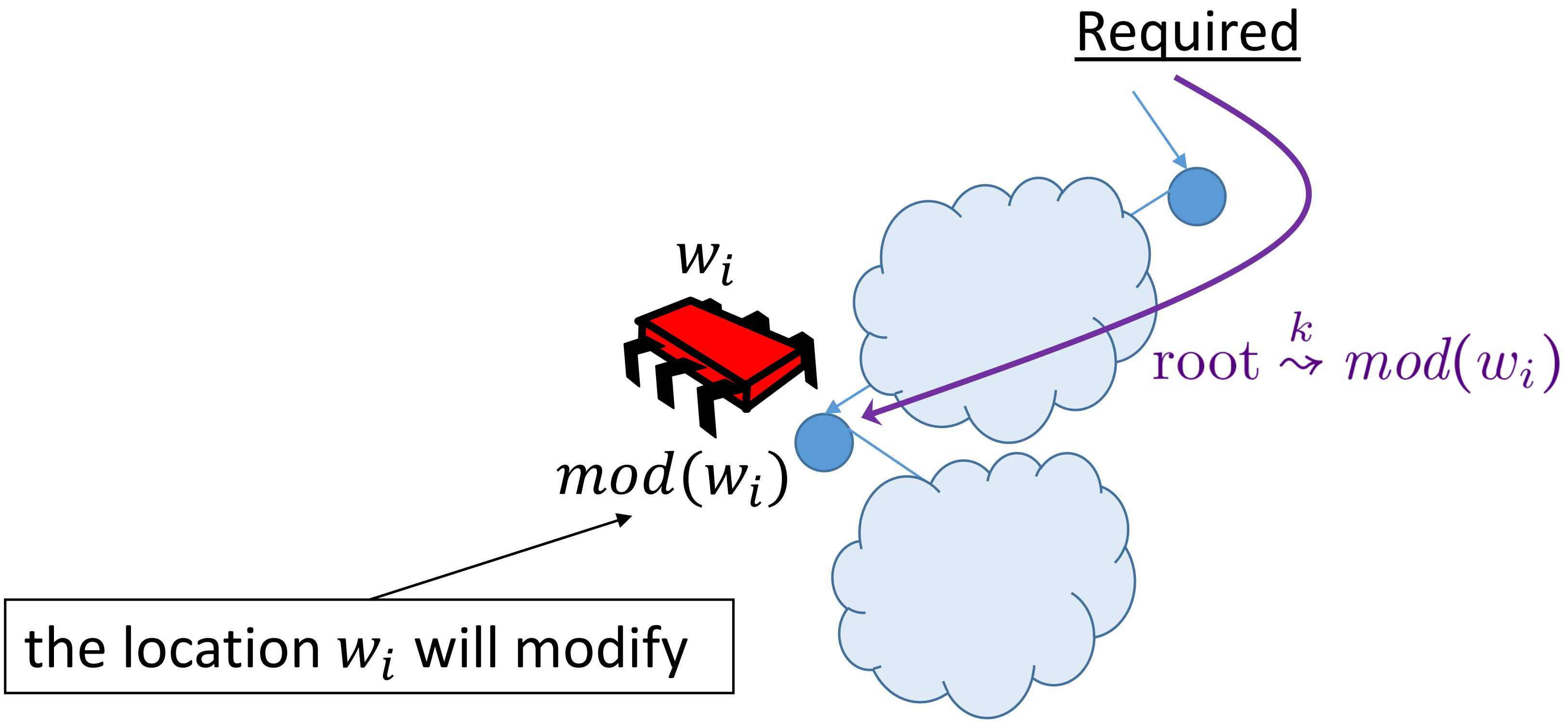


Requirement 2: Preservation

Writes do not destroy search paths to locations to which there could be later writes

Required

In example: nodes are **marked removed** when their **search reachability is reduced**;
writers lock and **check the marked** bit

$\overset{k}{\rightsquigarrow} mod(w_i)$

$mod(w_i)$

the location w_i will modify

Local View Argument

Order & Preservation

Sequential reasoning to show traversal correct without interference

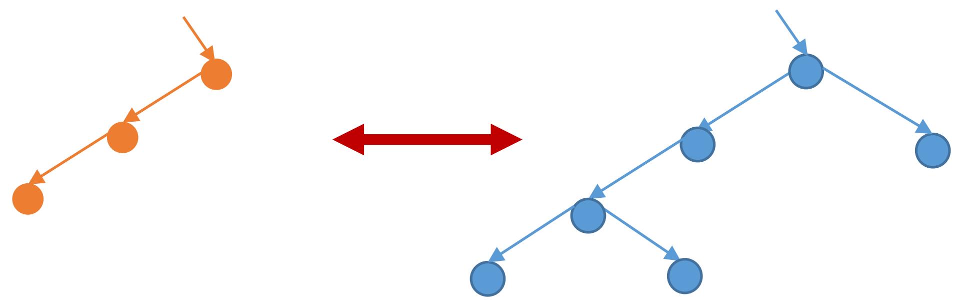
Correctness of concurrent traversals

$$\mathrm{root} \overset{k}{\rightsquigarrow} x \quad \Longrightarrow \quad \diamondminus(\mathrm{root} \overset{k}{\rightsquigarrow} x)$$

Idea: Consistency from Ordering

- Run with no interference on a related heap

thread's view

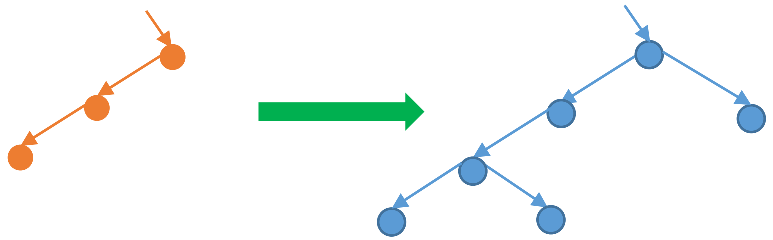


global memory

$H_r = w_{i_1} \ldots w_{i_k}(H_0)$ $\quad$ $H_c = w_1 \ldots w_n(H_0)$

- Writes on H_r are forward-agreeing with H_c
- Preservation complements the backwards path
- Full details in the paper

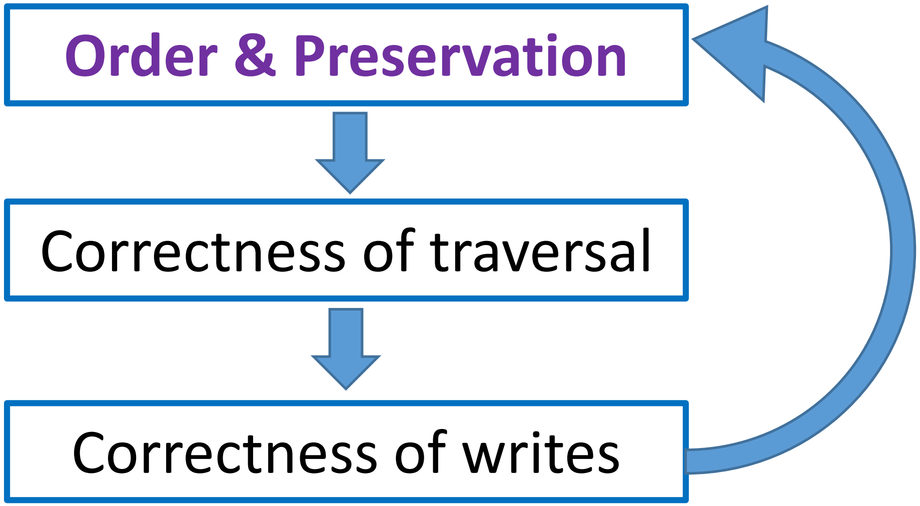
Summary



- Concurrent traversal correctness: Existence of search path in the past
- **Order** & **preservation** facilitate proof by **sequential reasoning**
- Reasoning on interleavings of writes completes the linearizability proof
- Challenges: backtracking, in-traversal validation

Establishing the Conditions

- In our examples, the proof of the conditions is trivial
- Valid to rely on local view arguments by induction on the sequence of writes



Some Related Work

- General linearizability proof methods and program logics – see paper for references
- Peter W. O'Hearn, Noam Rinetzky, Martin T. Vechev, Eran Yahav, Greta Yorsh:
  Verifying linearizability with hindsight. PODC 2010: 85-94
    - Proof of the lazy list by the hindsight lemma
    - Specific instance of our extension to path+field
- Kfir Lev-Ari, Gregory V. Chockler, Idit Keidar:
  A Constructive Approach for Proving Data Structures' Linearizability. DISC 2015: 356-370
    - Uses sequential reasoning on the data structure
    - Relies on **base points** – which our technique can establish
- Dennis E. Shasha, Nathan Goodman:
  Concurrent Search Structure Algorithms. ACM Trans. Database Syst. 13(1): 53-90 (1988)
    - Framework for proving linearizability of search data structures
    - 3 templates for traversal correctness, proofs require concurrent reasoning on interleavings of reads and writes

References

- [Fraser-04] Keir Fraser:
  Practical lock-freedom. PhD thesis
- [OPODIS’05] Steve Heller, Maurice Herlihy, Victor Luchangco, Mark Moir, William N. Scherer III, Nir Shavit:
  A Lazy Concurrent List-Based Set Algorithm. OPODIS 2005: 3-16
- [PODC’10] Faith Ellen, Panagiota Fatourou, Eric Ruppert, Franck van Breugel:
  Non-blocking binary search trees. PODC 2010: 131-140
- [SPAA’12] Shane V. Howley, Jeremy Jones:
  A non-blocking internal binary search tree. SPAA 2012: 161-171
- [EuroPar’13] Tyler Crain, Vincent Gramoli, Michel Raynal:
  A Contention-Friendly Binary Search Tree. Euro-Par 2013: 229-240
- [PPOPP’14b] Trevor Brown, Faith Ellen, Eric Ruppert:
  A general technique for non-blocking trees. PPOPP 2014: 329-342
- [PPOPP’14a] Aravind Natarajan, Neeraj Mittal:
  Fast concurrent lock-free binary search trees. PPOPP 2014: 317-328
- [ICDCN’15] Arunmoezhi Ramachandran, Neeraj Mittal:
  A Fast Lock-Free Internal Binary Search Tree. ICDCN 2015: 37:1-37:10
- [PPL’16] Tyler Crain, Vincent Gramoli, Michel Raynal:
  A Fast Contention-Friendly Binary Search Tree. Parallel Processing Letters 26(3): 1-17 (2016)