Homework number 3.

Final Research Project: Think about a research project and schedule a meeting with me to discuss it. Prepare a short summery of the research idea (less than a page). You need to define an *interesting research question*, and say why do you think it is interesting.

Question I: Consider the model of Herding, were a sequence of agents need to make a decision whether to to adopt or reject a project, based on their own signals and the decisions of previous agents. The project has a payoff $V \in \{0, 1\}$ (with equal probability), and a cost to adopting $C = 1/2$. Each agent observes a signal of $X_i \in \{H, L\}$, where H is observed with probability $p > 1/2$ if $V = 1$, and with probability $1 - p$ if $V = 0$.

Assume that there are *noise agents* that decide based only on their signal.

If every kth agent is a noise agent, what will be the observed behavior of the agents. (Assume $k >> 2$).

Question II: In the sponsored search model with budgets, assume that the budget of advertiser i increases with the number of impression it has so far. Namely, the budget at time t is $B_i + X_{i,t}$, where $X_{i,t} = \sum_{j=1}^{t} x_{i,j}$ and $x_{i,j}$ is 1 if advertiser i receives ad j and zero otherwise. (The bids $b_{i,j} \in [0, M]$ where $M \geq 1$.)

Design an online algorithm for this new model.

Question III: A resource allocation market has:

1. n agents with budget B_i.

2. There is a set R of resources, and each resource $r \in R$ has a quantity of c_r.

3. Each agent i has subsets $S_{i,1} \dots S_{i,k_i} \subset R$. Each subset $S_{i,j}$ can be used to produce a unit of object, given a unit of each of the resources in $S_{i,j}$. Each agent would like to maximize the number of objects it produces, given their budget constraint. (Namely, the agents buy resources in order to produce objects.) One can also produce fractional unit.

Show a convex program that derives market equilibrium, and show that in equilibrium:

1. Resource $r \in R$ has a positive price iff it is completely used.

2. Each agent uses only the cheapest sets to produce the objects

3. The budget of each agent is completely used up.

The homework is due in two weeks