

## Analysis of Recall and Recognition in a Certain Class of Adaptive Networks

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**Abstract.** A general model of adaptive networks which perform recall is analyzed in view of qualitative psychological findings. The assumptions underlying the basic model are few and general in the sense that no specifications of structure or mechanisms of adaptation are imposed. The analysis of the model is towards the addition of various features drawn from the global input-output relations expected from the networks. The memorizing process of recognition is found to be intrinsic to the model, and four of the most prominent relations between the performance of recall and recognition are shown to either exist in model or to be realizable by means of few additional plausible features.

### 1. Introduction

Adaptive networks of neuron-like elements are frequently employed in models of elementary learning and memory processes. Many such models have been proposed, differing in their structure, logic and performance. The study of such models can be aimed at two, not mutually exclusive, goals:

(i) The refinement of the properties of the basic elements according to physiological findings, and the identification of the network with real elements of the CNS — the “physiological approach”.

(ii) The construction of models consistent with global input-output relations, namely with results drawn from experimental psychology — the “psychological approach”.

The main disadvantage of the first approach is due to the extreme difficulty underlying any attempt to validate such models by neurophysiological experiments. This difficulty stems from the inability to record simultaneous numerous minor changes in numerous sites of the CNS.

In this work, we adopt the psychological approach and analyze a certain class of learning and memory models. Using psychological findings concerning the relations between the two basic memorizing processes of recall and recognition, we deduce various features of the network model.

### 2. The Network Model

We define here a generalized model based on a small number of specifications, representing a class of models with similar performance of learning. The model is an adaptive network of neuron-like elements (“cells”), capable of “learning”: having received a certain input several times, the network is changed so as to respond to a fraction of this input as if the whole input is being received. The model might operate in a discrete time scale, like models where the formal neuron of McCulloch and Pitts [1943] is employed, [Brindly, 1969; Fukushima, 1973], as well as in a continuous time scale [Wigstrom, 1973; Fukushima, 1975].

Our network consists of three main levels (a level may contain several cells layers): input, processing (or association) and output levels.

The specific structure of the input and output levels is not very essential to our analysis. For the sake of convenience, we assume the existence of an alphabet of elementary “words”, and a mapping of the alphabet onto the input cells, such that the number of input cells is linear in the alphabet size.

A certain set of input cells is fired by the input of its corresponding word. Each input cell is randomly connected to some of the association level cells. The input of a word sequence  $I$  activates some input cells, which activates, in their turn, a characteristic set of cells  $C(I)$  of the association level (see Fig. 1).

Since the connection between the two levels are random, the size of  $C(I)$  is a random variable with a

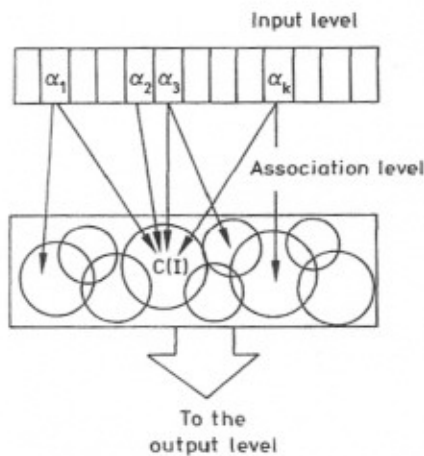


Fig. 1. A schematic outline of the model.  $I = \alpha_1, \alpha_2, \dots, \alpha_k$  is a  $k$ -word sequence. The corresponding input cells are connected to  $C(I)$

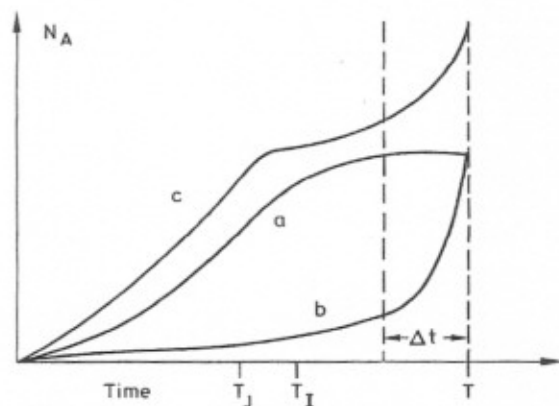


Fig. 2. Number of active cells in the association level ( $N_A$ ) vs. time, during the input of a word sequence  $I$ . The sequence  $I$  is (a) well learned, (b) not learned (c) not learned, but has the same first part as a well learned sequence  $J$

given distribution depending on the network structure and parameters. We assume that for  $I \neq J$ ,  $C(I) \neq C(J)$ , but not necessarily  $C(I) \cap C(J) = \emptyset$ , i.e. the characteristic sets are unique, but not disjoint. The cells of the association level are connected to the output level, and the activity of  $C(I)$  gives place to a certain output pattern.

The adaptability of the network is manifested by the following property: the part of  $C(I)$  that fires in response to a part of  $I$  is a nondecreasing function of the size of the received part of  $I$ , and of the number of times the full sequence  $I$  has been repeated. Striving for generality, we do not specify the detailed mechanism accounting for such an adaptability. Yet a possible mechanism, common to many models, assumes changes in the influence of one cell on another, by means of variable synaptic transmittance efficiency.

This idea of modifiable synapses was originally introduced by Hebb [1949], and has since been extended in several modes (exhaustively described by Fukushima [1975]).

Due to the random connections between the levels and the overlap in the characteristic sets, erroneous responses of the network are expected. In the sequel we analyze the performance of the model qualitatively, i.e. we deal with relations among the performance of various tasks, in view of psychological results, rather than with the numerical values of the corresponding error probabilities. This approach is adopted since any detailed error analysis is, to a large extent, model specific. (A typical example of such an analysis can be found in the work of Brindly [1969]).

### 3. Recall and Recognition in the Network Model

As is commonly defined, recall is the process of memorizing a currently non-presented item, with or without given clues, while recognition is the process of memorizing a presented item [Woodworth, 1949].

According to this definition most of the models mentioned in §2 perform recall. A basic assumption behind our analysis is that there is no multiple "storing" of the same information for different tasks, although there might be redundancy in representation. Using computer concepts (without making however any analogy between computers and the CNS), this hypothesis implies the existence of a unique "data base" accessible by several routines. Therefore the information stored in the network during the process of learning, must be sufficient for both recall and recognition of the learned word sequences.

#### 3.1. Recall Performance

A successful recall of a word sequence  $I$  from a part of it occurs when the received part of  $I$  excites enough cells of  $C(I)$  sufficient for the excitation of the output cells corresponding to the whole of  $C(I)$ .

Let  $n$  be the number of times the whole sequence  $I$  has been presented to the network, and let  $f_i(n)$  be the number of  $C(I)$  cells that fire after a certain (fixed) part of  $I$  is received. By the basic assumption of §2,  $f_i(n)$  is a nondecreasing function of  $n$ . A successful recall will take place, then, if  $f_i(n)$  is sufficiently large.

#### 3.2. Recognition Performance

The performance of recognition by the network is not as self evident as recall is, yet it is intrinsic to the model and is revealed by a more fine analysis. During a serial representation of a word sequence  $I$  in the time

interval  $[0, T]$ , an increasing number of cells of  $C(I)$  are excited. By the assumption of §2, in time  $t=T$  the whole  $C(I)$  is active. Let us consider the number of active cells in  $C(I)$  as a function of time for  $0 \leq t \leq T$ , in the following three exclusive situations:

(a)  $I$  is a well learned sequence; there exists a "critical time"  $T_I$ , such that for  $t=T_I$ , the already received part of  $I$  (termed hereafter the "critical part" of  $I$ ) is sufficient to evoke a response in almost all the cells of  $C(I)$  (Fig. 2a).

(b)  $I$  is an unlearned sequence; for  $t \leq T$ , an increasing number of  $C(I)$  cells become active, until the whole of  $C(I)$  is active, at  $t=T$  (Fig. 2b).

(c)  $I$  is an unlearned sequence, but there exists a learned sequence  $J$ , such that its critical part coincides with the first part of  $I$ . Thus, at time  $t=T_J$ , most of the cells of  $C(J)$  are active, while at  $t=T$  all  $C(I)$  becomes also active (Fig. 2c). Notice, however, that  $C(I)$  and  $C(J)$  are not necessarily disjoint, and therefore the increase in activity in  $[T_J, T]$  may be less than the size of  $C(I)$ .

A direct conclusion from this analysis is that discrimination between learned and unlearned sequences can be drawn solely from the temporal pattern of activity of the cells in the association level. A significant increase in activity during the end of the interval  $[0, T]$ , namely in  $[T-\Delta t, T]$ , where  $\Delta t$  is model specific, characterizes an unlearned sequence, while an almost stationary activity in  $[T-\Delta t, T]$  characterizes a learned one.  $\Delta t$  must be smaller than  $T-T_I$  in order that a perfectly learned sequence be recognizable. Obviously one cannot exclude the existence of sequences with  $T-T_I < \Delta t$ . Such sequences are not recognizable and in fact cannot be recalled since their critical part is almost the whole sequence.

Let  $g_I(n)$  denote the temporal change in activity in the association level during  $[T-\Delta t, T]$  for a sequence  $I$  which has been repeated  $n$  times. By the basic assumption of §2  $g_I(n)$  is a nonincreasing function of  $n$ . Thus the sequence  $I$  is recognized as an old one if  $g_I(n)$  is sufficiently small, and as a new one of  $g_I(n)$  is sufficiently large.

It should be noted that although  $f_I(n)$  and  $g_I(n)$  are sequence dependent, yet for any concrete model with explicit specifications of the random connections and the mechanism of adaptivity,  $f(n)$  and  $g(n)$  can be considered as random variables defined on the space of all possible sequences, with a distribution function determined by the model structure and parameters.

#### 4. Psychological Findings

The psychological findings relevant to the analysis of our model are concerned with the relations between

the two memorizing processes of recall and recognition. The following four observations [Kintsch, 1970] serve as guidelines to the "global" behaviour expected from the network model:

I. The processes of recall and recognition are both "threshold processes", in the sense that both involve a "decision threshold", above which a response is produced (recall), or an item is declared to be familiar (recognition).

II. It seems that recall and recognition, in spite of both being threshold processes, are two psychologically distinct processes. The main difference is that recall involves a "search" process, while recognition does not.

III. The most prominent observation in recall-recognition experiments is that the probability of a correct recognition is higher than the probability of a correct recall for items with the same number of repetitions. (See also Woodworth, 1949; Postman and Keppel, 1969).

IV. The frequency of an item in a "language" effects these two modes of memorizing in opposite directions; the more frequent an item is, the probability of recalling it increases, while the probability of recognizing it decreases [Kintsch, 1970].

Most of the above findings were obtained by what is called "free recall" experiments, i.e. recall of items from a repeatedly presented list without any given clues. This is not exactly the situation in our network model, where recall is initiated by some stimulus. Nevertheless, we shall use these four qualitative results as working hypotheses, assuming their validity in more general situations of recall and recognition.

#### 5. The Network Performance in View of the Psychological Findings

##### 5.1

The threshold property of the processes of recall and recognition is naturally incorporated into the network model, since both processes depend upon levels of activity. Let us introduce two network specific thresholds,  $\theta_1$  and  $\theta_2$ , such that for a sequence  $I$ , repeated  $n$  times, a successful recall occurs if  $f_I(n) > \theta_1$ , and a successful recognition occurs if  $g_I(n) < \theta_2$ .

The determination of  $\theta_1$  must be such that the probability of error in recall is minimal. Although as  $\theta_1$  decreases the probability of not recalling a learned sequence decreases, yet the probability of recalling an unlearned sequence increases. Therefore  $\theta_1$  has to be chosen according to some optimality criteria (e.g. minimizing the sum of the above mentioned two types of error probabilities). A similar argument applies to the determination of  $\theta_2$  [Yeshurun, 1977].

## 5.2

The main difference between the processes of recall and recognition, as described in §3, is that while recognition depends only upon the temporal activity pattern of the whole association level, recall depends upon the activity of specific subsets of the association level. Moreover, the activity in the association level following the representation of a part of a sequence  $I$  might be ambiguous, in the sense that the condition  $f_i(n) > \theta_1$  is simultaneously satisfied by several different characteristic sets. Such a situation can be attributed to several factors, among which the most significant ones are: ambiguous input, overlapping characteristic sets and background "noise". Thus, in the case of ambiguous activity, a choice must be made among the possible responses. This could be done by a search for that characteristic set with the maximal activity.

To summarize, recall depends upon the identity of the active set of cells, and often involves an explicit search process. Recognition, on the other hand, does not involve any kind of search.

## 5.3

The ability of the network to recall a well learned sequence  $I$  is equivalent to the existence of a minimal part of  $I$  (the initial part) which excites almost all the cells of  $C(I)$ .

The changes in the network in the course of "learning" by repetitions can take place either gradually or at once. In case of learning in the "all or none" fashion (Fig. 3b), the change in activity in the association level during  $[T - \Delta t, T]$  is small only when the network is already capable of recall. On the other hand, in the case of "incremental learning" (Fig. 3a), there is a considerable range of values of  $n$  for which the change in activity during  $[T - \Delta t, T]$  is small and recognition is possible, while for these values of  $n$  the critical part of  $I$  is not yet sufficient for its recall. Thus the requirement that recognition be superior to recall rules out the "all or none" mode of adaptation, and is consistent with the mode of incremental learning. This mode of learning can be realized in the network if adaptation is the result of the accumulation of many minor changes, occurring with a positive probability following each repetition.

Moreover, if the network is tested in an "experimental situation" where the clues in the recall experiments are restricted to an a priori given fixed fraction of the sequences (e.g. half of each sequence), then there might be "well learned" sequences with  $T - T_I > \Delta t$  but with a critical part greater than the allowed stimulus. Such sequences cannot be recalled but can be recognized.

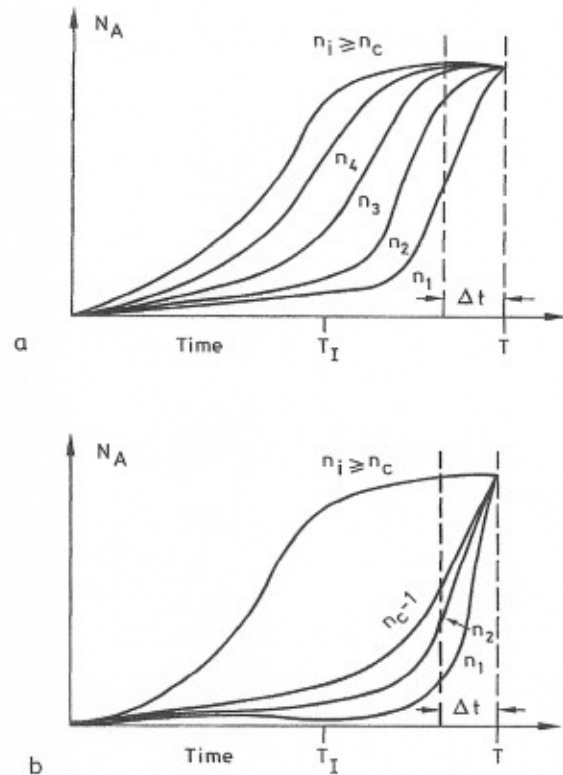


Fig. 3a and b. Number of active cells in the association level ( $N_A$ ) vs. time, during the input of a word sequence having been repeated  $n_i$  times  $n_1 < n_2 < \dots < n_c$ . a Learning is incremental; b learning is "all or none"

In the special case of  $n$ -word sequences which are to be recalled from their  $n-1$  first words [Brindley, 1969],  $\Delta t = T - T_I$ , and recognition is no more superior to recall. The expected superiority is retained, if there are modifiable connections between the association level and the output level, effecting the performance of recall but irrelevant to the performance of recognition [Yeshurun, 1977].

## 5.4

Property IV of §4, formulated in terms of the network parameters, is the following: for a fixed value of  $n$ , as the corresponding sequence is less frequent in the "language",  $P_r[f(n) > \theta_1]$  decreases while  $P_r[g(n) < \theta_2]$  increases.

To meet this requirement we further assume that:

(a) For a fixed value of  $n$ ,  $f_I(n)$  and  $g_I(n)$  are proportional to the number of cells in the characteristic set  $C(I)$ , namely

$$f_I(n) = |C(I)|\eta(n), \quad g_I(n) = |C(I)|\zeta(n)$$



where  $|C(I)|$  is the size of  $C(I)$ , and  $\eta(n)$ ,  $\zeta(n)$  are two random variables with values in  $[0, 1]$ , characterizing the mechanism of adaptation in the network.

(b) Characteristic sets corresponding to less frequent sequences tend to be smaller.

Then for equally learned sequences  $I$  and  $J$ , where  $I$  is less frequent than  $J$ ,  $|C(I)| < |C(J)|$  according to (b), while according to (a)

$$P_r[f_I(n) > \theta_1] = P_r\left[\eta(n) > \frac{\theta_1}{|C(I)|}\right] < P_r\left[\eta(n) > \frac{\theta_1}{|C(J)|}\right] \\ = P_r[f_J(n) > \theta_1]$$

$$P_r[g_I(n) < \theta_2] = P_r\left[\zeta(n) < \frac{\theta_2}{|C(I)|}\right] > P_r\left[\zeta(n) < \frac{\theta_2}{|C(J)|}\right] \\ = P_r[g_J(n) < \theta_2].$$

The detailed justification of assumptions (a) and (b) is beyond the scope of our analysis, yet we can outline here a general rationale. The plausibility of (a) stems from the random connections in the network and the probabilistic nature of the adaptation mechanism; for if all the elements of  $C(I)$  have an equal probability to be modified following each repetition, then  $f_I(n)$  and  $g_I(n)$  are indeed proportional to  $|C(I)|$ . The rationale (b) is more speculative in nature, and involves the initial formation of the connections within the network prior to the stage of "learning" of sequences. Suppose that before exposure to the "language", each input cell is connected to all the cells of the association level, and that during the early stages of its operation the more active connections are reinforced while the less active ones degenerate. If this is the case, then the final configuration may be of the requested type, namely larger characteristic sets for more frequent input sequences. This type of connection formation is analyzed in models of "self organization" (e.g. in von der Malsburg, 1973).

## 6. Summary and Discussion

The model analyzed in this work is an adaptive network of neuron-like elements ("cells") capable of performing the two basic memorizing processes of recall and recognition.

Three basic assumptions determine the structure of the model and its performance in general terms, without specifications of actual connections between and within levels, nor mechanisms of adaptation:

(1) There is an *input level* mapped one to one on an alphabet of "words".

(2) An *association level* is connected to the input level. The activity of the input cells corresponding to word sequence activates a "characteristic set" of cells in the association level. Different characteristic sets are not mutually disjoint.

(3) The network is capable of "learning" sequences by repetitions; after many "representations" of a sequence to the input level, the same characteristic activity in the association level can be activated by presenting the network with only a certain part of the sequence – the "critical part".

This performance of the network is realized by assuming that the number of active cells in a characteristic set is a monotone increasing function of the size of the part of the corresponding sequence presented to the network, and the number of times the sequence has been repeated before.

In our analysis the number of active cells measures levels of activity in a set of cells. This measure can be replaced by any other, e.g. the number of firings per unit time in a set, without effecting the conclusions.

With these three underlying assumptions we were able to show that:

(1) The network is capable of performing recall and recognition.

(2) Four well accepted relations between the performance of recall and recognition in humans (as found by psychological experiments), either exist in the network performance or can be naturally incorporated.

Our analysis leads to the following conclusions:

I. A successful recall occurs when a part of a "well learned" word sequence activates almost all of its characteristic set. A successful recognition occurs when during the representation of a sequence there is a small change in activity in the association level due to the information contained in the last part of the sequence.

Since both processes depend upon levels of activity being large enough or small enough, the psychological finding that recall and recognition are both "threshold processes" is natural to the model. Two thresholds (their values being model-specific) are introduced, the one is a lower bound to the amount of activity necessary for a successful recall and the other is an upper bound to the allowed change in activity for a successful recognition.

II. The process of recall in the network depends upon the identity of the active set of cells in the association level, while recognition depends on the temporal pattern of activity in the association level as a whole. Therefore recall is a search process while recognition is not, in agreement with the psychological findings.

III. The superiority of recognition to recall in case of learned sequences is a well established relation. It is realized in the model if the adaptation of the network due to repetitions of a sequence is achieved by the accumulation of many minor changes. This mode of adaptation is consistent with the notion of "incremental learning".

IV. The experimental observation that the more frequent an item is in the language, the probability of recalling it increases, while the probability of recognizing it decreases, is shown to be valid in our model, if two more features are introduced. Thus it is further required that

(a) larger characteristic sets correspond to more frequent word sequences,

(b) the number of cells, activated by the critical parts of equally learned sequences, is proportional to the size of the corresponding characteristic sets.

The basic assumptions underlying the model, together with the additional features required by our analysis, are shown to be plausible in the sense that networks with these features can be constructed according to physiologically accepted principles. To be more specific, our model is consistent with random connections within the network, uniformity of properties of neurons in the same functioning level, adaptability due to accumulation of many minor modifications, "self organization" in connection formation, information expressed in terms of the temporal or spatial pattern of activity of an aggregate of neurons, and the use of the same stored information for different tasks.

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Received: September 29, 1978

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